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DEBT SUSTAINABILITY ANALYSIS IN PAKISTAN: A FAN CHART APPROACH

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Abstract

This study analyzes public debt sustainability for next eight years for Pakistan using fan chart approach. The study is divided into three blocks. In first block, fiscal reaction function is used to test debt sustainability for 1984-85 to 2016-17 periods. In second block, non- fiscal determinants of debt are analyzed through unrestricted VAR model for the period 1984 (Q_1) to 2017 (Q_2) and their projections for next eight years are derived. While, in third block, VAR projections and estimated fiscal reaction function are combined to calibrate 1000 debt paths for each year of simulation and then fan charts for debt are drawn. These fan charts summarize the magnitude of risk, uncertain economic conditions and endogenous response of fiscal policy through diagrammatic representations of large sample of debt paths. The study finally concludes that the debt level is not sustainable in Pakistan.

Keywords: Debt Sustainability, Fan Chart, Approach

JEL Classification: F34; E60; E63

1. INTRODUCTION

Growing public debt is a serious contemporary issue worldwide. Public debt is a significant source of credit owed by the government to affirm growth and development of the country, which is not possible with the present domestic resources (Ahmed and Ahmed, 1998). Further, debt is borrowed by countries to finance development projects, to buy equipment etc. (Chaudhary & Shabbir, 2005; Akram, 2011). The government can borrow debt directly from the central bank, non-banking sector, commercial banks and also from other financial organizations like IMF, World Bank, and a foreign government, etc. The government repays interest and principal on the debt at regular intervals. The public debt includes both domestic public debt and external public debt and has both positive and negative effects on the economy. On one hand, it has a positive effect on economic growth and development when the government borrows to increase capital inflow, to provide foreign capital for industrial development, and to use expenditures for growth (Zaman and Arslan, 2014). On the other hand, it has a negative effect on economic growth and development when the fiscal deficit prevails long in an economy and public debt reaches beyond a certain limit that reduces the level of investment, raises unemployment and poverty, slows export, increases corruption and taxation (Islam and Biswas, 2005; Khan, 2016).

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In Pakistan, public debt has grown rapidly at a high rate during the last sixty years according to the Pakistan Economic Survey (2015-16). The issue of unsustainable public debt and persistent fiscal deficit is a serious problem in developing countries like Pakistan. In Pakistan due to high expenditures and low revenue growth, the public sector is compelled to increase its debt to pay for its expenditures which results in increasing fiscal deficit (Hussain & Idrees, 2015). The rate at which the government borrows debt is high enough to pay back its liability in an orderly manner and it transfers the burden to our coming generations that may be considered as a serious threat to the progress of country in future (Khan, 2016). The public debt mismanagement and limitless expenditure of government may result in negative trends in macroeconomic variables like the decline in non-debt creating inflows as well as a decrease in exports, decrease in investment, increase in inflation rate and variation in the exchange rate, etc. (Bilquees, 2003; Ejaz & Javid, 2011;Mushtaq & Zaman, 2013; Mirza & Mushtaq, 2015). In addition, the indirect consequences of rising debt are like macroeconomic instability, increase in unemployment and poverty as well as increase in uncertainty and decrease in investment posing a threat to economic progress and growth. (Khan, 2016)

According to World Bank Report [2000, 2001], developing countries like Pakistan are facing serious debt issues. In addition, the Fiscal Responsibility and Debt Limitation [FDRL] Act of 2005 mentioned that the total threshold level of Pakistan's debt should be 60% of gross domestic product (GDP). While Debt Policy Statement (2017-18) under the Ministry of Finance stated that at the end of June 2017, the public debt of Pakistan was 67.2% of GDP that is above the original limit and may be harmful in the long run. Therefore, in such a premise, it is highly significant to study public debt sustainability for Pakistan's economy in detail. Most of the previous works use a deterministic approach like "bound testing" for debt sustainability analysis (Apergis, 2000; Getzner et. al., 2001; Baharumshah, 2003; Afonso, 2003; Mirza & Mushtaq, 2015). This approach to risk assessment has some drawbacks which are limited to shocks that usually ignore the co-movements among macro variables. It also uses exogenous fiscal behavior in which fiscal policy does not respond to its own residuals and simulated economic shocks, which usually ignores endogenous factors which lead to false assessment of risk (Abiad and Ostry, 2005). The measurement of risk to debt sustainability demands a stochastic simulation algorithm that improves risk assessment of debt dynamics. Therefore, this study utilizes a stochastic simulation algorithm which consists of an endogenous fiscal reaction function to study fiscal policy response to simulated economic shocks which reacts against some macroeconomic changes effectively (Abiad and Ostry, 2005; Afonso, 2003; Greiner et. al., 2007).

There are many studies including Ejaz and Javid (2011), Mahmood and Rauf (2012) Chandia and Javid (2013), Mahmood et. al., (2014) on the topic regarding debt sustainability in Pakistan but they have used different methodologies to deal with this topic. Therefore, this study applies the probabilistic approach following the methodology of Celasun et. al. (2006; 2007) to investigate debt sustainability

in Pakistan for the period 1984- 2017. There is no study on debt sustainability analysis which is based on probabilistic approach to draw fan charts in Pakistan. This analysis is helpful to improve the common debt sustainability analysis by obtaining the endogenous response to fiscal policy which is the main contribution of this study. This study also captures patterns of shocks in terms of the frequency distribution to describe effectively the extent of risk because of uncertain economic conditions. Fan charts show a better reflection of the country's current fiscal policy position which is helpful to guide policymakers in forming their policies effectively. The study is formed into 6 main sections. Section 2 presents a literature review of the study. Section 3 presents an overview of the debt profile of Pakistan. Section 4 provides the methodology of the econometric model and sources of data. Section 5 presents empirical results and its interpretation. Section 6 briefly presents conclusions and some policy implications.

2. LITERATURE REVIEW

A review on debt sustainability analysis by using time series methods is firstly provided by (Heinemann, 1992). All the necessary conditions of debt sustainability are defined by Domar (1944) which is based on the approach of Keynes. The literature presents various approaches to explain the sustainability of fiscal policies. Some studies used probabilistic approach to examine public debt sustainability analysis (DSA) and also employed fiscal reaction function and unrestricted VAR model to investigate the risk factors (Afonso (2003), Abiad and Ostry (2005), Makin (2005), Celasun et. al. (2006), Penalver and Thwaites (2006), Garcia and Rigobon (2004; 2005). The literature also presents how Fiscal Reaction function is used to capture the endogenous fiscal policy response which reacts against some macroeconomic factors.

Many studies following this approach including (Mello, (2006); Penalver and Thwaites, (2006); Celasun et. al., (2006),; Khalid et. al., (2007), Afonso, 2009; and Burger et. al., (2011), Nguyen, 2013, Mutuku, 2015 to empirically examine the fiscal behavior and its determinants. A key query is whether the estimated fiscal reaction function is linear or not. Most of studies use linear fiscal reaction function to consider some macroeconomic changes effectively and that assures sustainability (Celasun et. al., 2006; Khalid et. al., 2007; Ejaz and Javid 2011). However, the studies that use non-linear (cubic) reaction function to investigate sustainability analysis are very rare (including Mendoza and Ostry, 2008, Ostry et. al., 2010 and Ghosh et. al., 2013).There are many recent studies which employ fiscal reaction function to capture fiscal policy behavior by employing different methods like (threshold regression models, autoregressive ordinary least squares, vector error-correction model, generalized least square (GLS) method (Khalid et. al., 2007; Mello, 2008; Shijaku, 2012; Burger et. al., 2012; Nguyen, 2013; Ghosh et. al., 2013; Chandia and Javid, 2013;Consiglio and Zenios, 2017).

The literature presents bundle of studies related to public debt sustainability and how to deal with fiscal deficit including (Ishfaq et. al., 1999; Chaudhary and

Shabbir, 2005; Bilquees, 2003; Mukhtar, 2007; jafri, 2008 and Siddiqi and Ilyas, 2011) mentioned that, there is significant and a positive relation among real exchange rate and budget deficit and conclude that high deficit makes huge debt that may incorporate extra load of debt in long term. Summing up the literature on analysis of public debt sustainability, most of the studies conclude that there is significant role of primary budget balance (deficit), current account balance, low growth rate of exports and many other factors contributing their part in forming debt levels of Pakistan and other developing countries unsustainable (Ejaz and Javid, 2011; Chandia and Javid, 2013; Ramzan et. al., 2013). Most of the studies conclude that increase in the total government expenditures increase the debt burden and adversely affect the debt sustainability condition (Mahmood et. al., 2014; Mirza & Mushtaq, 2015, and Hussain & Idrees, 2015).In traditional methods for investigating debt sustainability is mainly considered "bound test" approach which has some drawbacks. First, this approach is totally deterministic to risk assessment, involving assumptions in which one main variable is hit by an inauspicious shock at one time including high interest, low growth rates, low primary balance, and exogenous increase of debt that affects level of debt-to-GDP ratio (Goldfajn, 2005). Second, it also ignores covariance among shocks and misleads risk assessment of debt sustainability analysis. Previously, a number of studies used "bound test" approachsuch (as Getzner et. al., 2001; Baharumshah et. al., 2003; Afonso, 2003 etc.).

The limitations mentioned above indicate that the measurement of risks to debt sustainability analysis demands a stochastic simulation method which is capable to attain pattern of shocks and measurement of risks and incorporates co-variances of these shocks. Therefore, Celasun et. al., (2006) suggest probabilistic approach to assess debt sustainability analysis which can generate large frequency distributions of the debt ratio for each year. These projections obtained through endogenous response of fiscal policy to debt shocks of important macro variables like interest rates, exchange rates and growth rate. This stochastic simulation algorithm attempts to fulfill three main objectives: first, it provides meaningful way to describe and seek factors responsible for changing fiscal behavior. Second, large numbers of random shock constellations are drawn from an estimated joint distribution through endogenous fiscal policy. Third, the probabilistic approach to fan charts is derived through frequency distributions of debt ratio that presents the patterns of risk and shocks of the fiscal policy behavior effectively.

Therefore, with this background, our study will follow the methodology of (Celasun et. al., 2006) with some modification as an under lying framework of accounting probabilistic approach to debt sustainability analysis for Pakistan. This gives a much nuance and credible judgment of debt sustainability analysis. This analysis is helpful in making decision for policy makers and fan chart approach is simple to obtain pattern of shocks in terms of the frequency distribution to describe the extent of risks because of uncertain economic conditions effectively.

3. HISTORY AND COMPOSITION OF PAKISTAN'S PUBLIC DEBT

The economy of Pakistan received lack of resources during partition from India. The domestic saving from internal resources was very less so Pakistan preferred external borrowing. According to the history of Pakistan from 1950 to 1960, the purpose of external borrowing was to increase growth rate of the economy. The study of Ahmad (1999) supported that the growth scheme in 1960s was productive because external borrowings was used in high return developmental projects which resulted in rapid economic growth rate. The Human Development Report (1956) stated that the aggregate amount of external debt approximated to $121 million in Pakistan. In 1970, the economy of Pakistan faced issue of fiscal deficit this led to decrease in growth rate but still the external debt was manageable in the economy. In 1971, the East Pakistan tragedy created trade deficit, due to lack of exports and more dependency on imports the economy of Pakistan was in need to increase external borrowing to pay its liabilities. In 1973, there was oil prices shock, which led to high inflation and recession increased the volume of international indebtedness as well as its debt servicing liabilities.

Further, in 1980s to support Afghan War, Pakistan borrowed large amount of external debt from America under SAP (Structural Adjustment Program), the public debt of Pakistan rose rapidly from Rs. 168 billion to Rs. 709 billion and foreign debt of Pakistan rose from Rs. 86.7 billion to Rs. 324 billion by the end of 90s. In 1990, the situation of debt was horrible because of greater dependency on SAP loans, the external debt of Pakistan grew rapidly due to privatization of industry and business, the World Bank and IMF granted loan of $400 million in SAP program. However, the quantity of loan remained low during 1998 to 2002 due to the economic embargo against nuclear deals and political instability. The external debt of Pakistan increased quickly from $2.7 billion to $66.243 billion to counter balance of payment crises from December 1969 to 2012 period which shows unmanageable condition of debt in a country.

The recent trend shows that the total public debt reached at Rs. 21.4 trillion at 2017 period which was almost equal to 67.2% of GDP which is violation of FRDL Act (2005) and also harmful for a developing country like Pakistan, a debt-to-GDP ratio below 60% threshold was considered. Since 1958, Pakistan made 21 agreements of loans with IMF to solve low foreign exchange reserves and a balance-of-payments crisis. A report of State Bank of Pakistan (SBP) stated that Pakistan needed USD 11.7 billion to service its external debt in the current fiscal year (2018-19). Instead to mobilize domestic resources to increase tax and to promote exports the successive governments emphasis was to release dollars through bailout programs. Due to growing public debt burden over the past years the fiscal sector of Pakistan is generally considered as one of the central weaknesses of the economy. Over past years, there has been growing change in the total debt and percentage share of domestic and foreign debt in total debt and we present it in Table 1.

Table: 1Trends in Domestic and Foreign Debt (2011-2017)

Years	Domestic Debt	Foreign Debt	Total Debt	Domestic Debt	Foreign Debt
	(Rs. in Billions)			Percentage Share in Total Debt	
1980-81	58.1	86.7	144.9	40.1	69.9
1985-86	203.1	179.2	38.24	53.1	46.9
1990-91	448.2	346.9	795	56.4	46.6
1995-96	920.3	748.3	1668.1	55.2	44.9
2000-01	1789.2	1496.4	3292.6	54.6	44.4
2005-06	2152.0	1913.0	4064.0	52.9	47.1
2010-11	4654.3	4351.9	9006.2	51.7	48.3
2015-16	12198.9	5181.8	17380.7	70.2	29.8
2016-17	12781.7	5424.1	18142.8	70.1	29.9

Source: Pakistan Economic Survey (2015-16)

It is clear from Table 1 that an increasing trend prevailed from 1980-81 to 2016-2017 which is an alarming situation for our economy. It is clear from Table 1 that domestic debt percentage share in total debt was less than 50 percent until 1980-84, due to positive savings rate. It increased from 51 percent in 1984-85 to 70 percent in 2016-17. The percentage of domestic debt exceeded the foreign debt as also presented in table and it is due to limited availability of external resources. However, foreign debt as a percent of total debt declined over time from 69.9 percent during 1980-81 to 29.9 percent in 2016-17, while domestic debt increased from 40 percent to 70 percent over the same period. History shows that since its independence there is growing change in the trends of debt level of Pakistan economy. The primary reason of growing debt burden is due to the large primary deficit and inability of political leaders to cut down fiscal deficit significantly. The rate of growth of real debt was substantially higher than the growth rate of GDP and exceeded growth of government revenues. Therefore, it is important for the government to adopt proper structural reforms and corrective measures to keep debt reduction which boost up the potential growth and ensure public debt sustainability. The government may adopt comprehensive approach necessary for fiscal consolidation and debt management to utilize debt properly which leads to higher economic growth, boosts exports and other foreign exchange earnings.

4. METHODOLOGY

This study investigates debt sustainability using probabilistic approach to draw fan charts by incorporating endogenous response of fiscal policy in this analysis Celasun et. al., (2006; 2007). Further, this study also includes other non-fiscal

factors responsible for debt to GDP becoming unmanageable and unsustainable. This section subdivided into three sections. The section 4.1 explains methodology for debt sustainability analysis and this section further consists of three blocks. First Block explains the fiscal reaction function. Second Block explains VAR model specification. Third Block explains simulation algorithm fan charts, and construction of debt sustainability indicator for debt-to-GDP ratio. Section 4.2 provides description of variables used in this model.

4.1. Methodology for Debt Sustainability Analysis

The comprehensive review of literature above shows that there are two fundamental grounds for using simulation algorithm of Celasun, et. al., (2006). First, it offers a valid method to deal fiscal behavior and imitate changes to it. Second, it applies easily to various developing countries like Pakistan to provide insights for policy judgments. According to the methodology of Celasun, et. al.,(2006) the forecast of the fiscal reaction function is kept apart from other economic relationships and combine them again when projecting the pattern of the public debt ratio. The reason behind this scenario is that the variance-covariance matrix of shocks ineluctably relies on an unrestricted VAR model using time-series which limits the specification of the reaction function in unwanted manners such as, the primary balance reacts to changes during same period in the output gap but not in the lagged variables for details see Celasun et. al., (2006).

Therefore, first block describes fiscal behavior through an explicit fiscal reaction function that is calibrated using time series data obtained for Pakistan during 1984-85 to 2016-17 periods. The reaction function allows endogenous fiscal policy responses that make risk analysis better by perceiving the sensible policy reaction of the primary balance to different economic impacts. In Second Block, the joint distribution of shocks of debt dynamics are captured through unrestricted VAR models. The unrestricted VAR model provides important functions: (i) It describes the co-movements of debt dynamics elements (like real GDP growth, real interest rates and real exchange rates); (ii) It gives estimates of conditional variances and co-variance matrix of the shocks; (iii) Generates a logical set of projections for the elements of debt dynamics by using quarterly data. The third block gathers the simulated economic assumptions of (second block) with the forecasted fiscal policy process in (first block) to obtain annual public debt series. Hence, with the repeated simulations of random shocks, we produce a large sample of public debt projections for each year of the forecasting horizon. The resulting frequency distribution shows a probabilistic appraisal of debt dynamics. Specifically, we draw “fan charts” which depict confidence bands for various degrees of uncertainty about the median projection, based solely on the trend of central projection for debt series. Now, we discuss these blocks in detail:

First Block: A Fiscal Reaction Function

In first block study estimates a fiscal reaction function for Pakistan economy by employing time series annual data from 1984-45 to 2016-17 as specified by equation 4. 1.

$$p_t = \alpha_0 + \rho d_{t-1} + \gamma ygap_t + \beta inf_{t-1} + \varepsilon_t \quad t = 1, 2, \ldots\ldots T \qquad 4.1$$

This equation shows the relationship of P_t that is the ratio of primary balance to GDP as dependent variable, the d_{t-1} which is lag public debt to GDP ratio as an independent variable, $ygap_t$ is the output gap (showing gap between actual and trend level of GDP) as another important determinants of primary balance and inf_t that is inflation which is captured by consumer price index this measure is also suggested by number of studies including Khalid et. al., 2007; Nguyen, 2013; Mutuku, 2015.

Second Block: Non-Fiscal Determinants of Public Debt Dynamics:

In second block this study estimates an unrestricted VAR model. The VAR model captures non-fiscal determinants of public debt dynamic in Pakistan. The VAR model describes two basic components of the stochastic simulation. First, it provides variance-covariance matrix of the residuals that shows the combine statistical properties of the contemporaneous shocks involving the debt dynamics. Second, the forecasted coefficients of the VAR model permit to produce projections that enter in the equation of debt, then obtain a central trends and alternative paths as well. The important variable used in the study for estimation of unrestricted VAR are key determinants of fiscal policy including the real domestic interest rate, real foreign interest rate, real exchange rate and real GDP growth rate. Symbolically the unrestricted VAR model is specified as follows:

$$Y_t = \gamma_0 + \sum_{k=1}^{P} \gamma_k Y_{t-k} + \xi_t \qquad \text{Where,} \quad Y = \left(r_t^{us}, r_t^d, gr_t, er_t\right) \qquad 4.2$$

γ_k is a vector of coefficients, r_t^{us} is the real foreign interest rate, r_t^d is the real domestic interest rate, gr_t is the real GDP growth rate, er_t is the real exchange rate, ξ_t is the vector of error terms such that ξ_t ~N(0,Ω) where, Ω is the variance-covariance matrix of residuals and it also summarizes the joint statistical properties of contemporaneous non-fiscal disturbances affecting debt dynamics (it is suggested by number of studies including Garcia and Rigobon, 2005; Celasun, et. al.,2006 and Burger et. al. 2011.

Then simulations use a sequence of random number $\hat{\xi}_{t+1}, \hat{\xi}_{t+2}, \ldots\ldots, \hat{\xi}_T$ such that τ [t+1, T]; $\hat{\xi}_\tau = W\upsilon_\tau$ where υ_τ~N (0, 1),and W is such that Ω =W′W (W is the choleski factorization of Ω. These simulations are repeated thousand times for each of next eight years. The VAR model describes the co-movements between the non-fiscal determinants of debt (Ω) including interest rates, exchange rates

and real GDP growth and obtains the elements of Y consistent with the simulated shocks. For each period, the VAR model develops combine dynamic responses of all factors in Y.

The simulation procedure used in this study is not sensitive to ordering of VAR variables because present study does not obtain causal relationships, it only provides joint dynamics for all the variables. This study derives stochastic simulation results through variance-covariance matrix of unrestricted errors Ω, which is unique (Garcia and Rigobon, 2005).Hence, VAR model is forecasted like a risk analysis approach, instead of using the coefficients of the VAR to obtain the central projection for the debt-to-GDP ratio, the VAR is used to obtain the variance- covariance matrix for the joint distribution of the errors.

Third Block: Debt Paths, Fan Charts and Sustainability Assessment:

In third block, for each constellation of shocks we annualize the quarterly projections of VAR and the simulated annual values of output gap are derived from the growth differential between predicted GDP growth and the (annualized) steady-state growth rate produced by VAR (to ensure that shocks to the output gap are zero on average). The respective debt paths are estimated by employing equation (4.1) and the conventional debt stock-flow identity presented in equation 4.3:

$$d_t = (1+g_t)^{-1}\left[\left(1+r_t^f\right)(1+\Delta z)d_{t-1}^{*} + (1+d_t)\tilde{d}_{t-1}\right] - p_t + s_t \qquad 4.3$$

Where, d_t^* is the foreign debt denominated in the domestic currency. $\tilde{d}_{t-1}$ is the domestic debt measured in domestic currency. s_t is the stock-flow modification due to the realization of contingent liabilities. p_t is the primary balance that hold a fiscal policy shock in each simulation, with σ^2 as the variance of the fiscal reaction function of residuals as suggested by Celasun et. al. (2006; 2007)..

We assume that all the simulations are based on same joint distribution of disturbances and co movements among variables. The fiscal reaction function blends Pakistan's fiscal policy characteristics. The upper "hat" represents parameter forecast in the first block:

$$\hat{p}_{t+\tau} = \Lambda_{t+\tau} + \hat{\rho}d_{t+\tau-1} + \hat{\gamma}ygap_{t+\tau} + \phi_{t+\tau} + \alpha\hat{p}_{t+\tau-1} + \beta\hat{p}_{t+\tau-2} \qquad 4.4$$

For τ =1…..8 and t = 2016

For instance, $\Lambda_{t+\tau} = \hat{p}_t - \hat{\rho}d_{t-1} - \hat{\gamma}ygap_t + \kappa_{t+\tau} + \hat{p}_{t-1} + \hat{p}_{t-2}$ and $\phi_{t+\tau}$, is the fiscal policy shock which is based on the distributions of shocks with mean zero and variance equal to variance of residual. The equation (4.4) above separate the fiscal policy into three parts i.e., i) predetermined part ii) automatic part iii) random part. The pre-determined portion is given by $\Lambda_{t+\tau}$, that shows the effect of all other elements of primary balances. $\phi_{t+\tau}$ is a random part and by default

we assume $\kappa_{t+\tau}$ =0 that captures future policy change. Later, we will change this assumption according to constant or predicted policy analysis. The automatic part shows the average response of primary balance to public debt, output gap and lagged primary balances.

After 1000 simulations we form frequency distribution for each year and use them to draw fan charts. Finally, we construct fan charts and a summary indicator of debt sustainability to analyze the risk analysis under two potentials "baseline assumptions." The baseline assumptions use in this study permits automatic reaction of the primary balance to output shocks and public debt series. The measures of $\Lambda_{t+\tau}$ comprises the residual from the reaction function over period t. The difference among predicted primary balance as well as actual fiscal behavior demands adjustment effort and response depends on whether discrepancy is permanent or temporary.

First is the "constant" policy assumption, every difference of the predicted primary surplus of t period presumes to stay same throughout full simulation period indicating that the most recent stance indicates persisting or sustained derivation from past behavior of primary surplus (in the case, $k_{t+\tau}$=0 for full simulation period). Such assumption fixes (non-debt relates) discretionary component of the policy. Second is "predicted" policy assumption, the recent difference of predicted primary surplus that result from exclusively temporary elements, it assumes that the simulated paths follow the predictions of reaction function (Therefore, $k_{t+\tau}= -\hat{\varepsilon}_t$ over full simulation horizon). In both events, we use common specification of linear reaction function for simulations and t = 2016-17 and τ = 8 for i.e., next 8 years for the economy of Pakistan. This algorithm yields randomly a large set of 1000 debt series matching to different shock constellations to obtain final public debt series. Afterwards, the corresponding frequency distribution of forecasted debt series is established for every year and used to derive fan charts graph and probabilistic sustainability indicator. Fan charts summarize risks to debt dynamics based on frequency distribution of a large sample of debt paths generated by means of stochastic simulations. Different colors are used to specify deciles in the distributions of debt ratios.

From frequency distribution we form probability distribution of debt. Based on these probabilities we form sustainability indicators as $\Pr(d_{t+\tau} > d_t) \times$[1-pr $(d_{t+\tau}>(d_t+x))$], where x is prescribed mark-up over initial public debt d_t. This indicator combines the probability that debt will decline overtime. The risk that debt will not rise beyond same level over the same horizon where, x is prescribed markup over initial public debt d_t.The sustainability indicator value increases with the favorable results, which is non-increasing trends and well controlled upside risks. That value has no inherent meaning and policymaker can adjust this critical threshold which usually relies on the quantity of risk aversion and on perceived need to decrease debt. These elements are reflected in the markup x, the level of upside risk considered acceptable (i.e., $\Pr(d_{t+\tau} > (d_t+x))$ and a

desire probability that will decrease in future. Once these probabilities in sustainability indicator are set, a critical threshold is established. The policymakers then focus about problematic events when the measure of the indicator decreases below the sustainability indicator's threshold level.

4.2. Data and Source

The availability of reliable data is very important before carrying out an empirical research. Otherwise, the use of incorrect data leads to incorrect conclusion. This study used annual data for fiscal reaction function from the period 1984-85 to 2016-17 and for the approximation of unrestricted VAR model quarterly data is used from period 1984 (Q_1) – 2017 (Q_2) for Pakistan. This study utilized secondary data and collects data from several sources for the estimation of fiscal reaction function. The important variables used in this study for estimation of fiscal reaction function are primary balance (as ratio of GDP), public debt (as ratio of GDP**)**, output gap and CPI based inflation. This study uses secondary data for unrestricted VAR model and collects quarterly data from International Financial Statistics (IFS). The important variables used for estimation of unrestricted VAR which are non-fiscal determinants of public debt of Pakistan including the real domestic interest rate, real foreign interest rate, real GDP growth rate, and real exchange rate. Table 2 provides brief description of variables and data sources.

Table: 2 Data and Variable Description

Variables	Definition	Source
Primary Blance (Ratio of GDP)	Primary balance (as ratio of GDP) as the dependent variable. The primary balance to GDP is government net borrowing after excluding the interest payments on consolidated government liabilities.	Handbook of Pakistan (2011).
Lagged Public Debt (Ratio of GDP)	Lag value of public debt as ratio of GDP as independent variable. It specifies a country's debt compare to its economic output.	Pakistan Economic Survey (2015-17).
Output Gap	Output gap as main variable use as control for the effects of the business cycle. It is difference between the actual and potential output of country, expressed as a percentage of gross domestic products (GDP)	Hedrick-Prescott Filtered Trend
CPI index	Inflation rate is evaluated by using the consumer price index (CPI). The CPI is evaluated as change in prices paid by consumers for the goods and services in market and we use 2010 as a base year.	Handbook of Pakistan (2011).
Real Foreign Interest Rate	We use call money rate of USA and convert it to real interest rate by using Fisher equation and use it as real foreign interest rate and 2010 as base year.	International Financial Statistics

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(continued) Table: 2 Data and Variable Description

Variables	Definition	Source
Real Domestic Interest Rate	We use call money rate of Pakistan and convert it to real interest rate by using Fisher equation and use it as real domestic interest rate. The domestic interest rate is used in foreign exchange markets for interest rate parity calculations and we used 2010 as base year.	International Financial Statistics
Real GDP Growth Rate	The growth rate of real GDP at which gross domestic product (GDP) of country changes from one year to another. The GDP is the market value of all goods and services produced in a country in a specific year.	International Financial Statistics

5. EMPIRICAL RESULT

This chapter demonstrates the result of debt sustainability analysis for Pakistan. First section 5.1 of this chapter presents the results of fiscal reaction function. Section 5.2 presents the results of VAR model as well. Finally, in section 5.3 we present fan charts. The frequency distributions of forecasted debt series are used to drive fan charts for Pakistan.

5.1. The Fiscal Reaction Function

The linear combination of variables in the fiscal reaction function is first difference stationary. Therefore, we estimate the fiscal reaction function as specified in equation 4.1 by OLS (Ordinary Least Square) using annual data for period 1984-85 to 2016-17. The fiscal reaction function is estimated by regressing primary balance as ratio of GDP on lagged of public debt to GDP ratio, output gap as percentage of GDP and rate of inflation. The policy changes can be predicted through economic indicators like output gap and inflation. The summary of results is presented in Table 3 below.

Table 3 Estimates of the Fiscal Reaction Function (1984-45 to 2016-17)

Dependent variable is primary balance as ratio of (GDP)

Variable	Coefficient	t-value
Lagged debt	0.1081**	3.3261
	[0.0325]	
Output gap	0.0009	0.5734
	[0.0015]	
Inflation	0.0002	2.5630
	[8.31]	
Constant	-0.1026	-3.7548
	[0.0273]	

*NOTE: Standard errors are in parenthesis. *, ** and ***indicate significance at 10%, at 5% and 1% level of significance, respectively.*

We start by estimating fiscal reaction function using primary balance as the fiscal policy instrument. The regressed equation shows positive and significant relationship of lagged debt with primary balances while coefficients of output gap and inflation appears insignificant. When we applied LM test, we come to know about the existence of autocorrelation problem. With the inclusion of first lag still we face the problem of autocorrelation. But, the inclusion of second lag of dependent variable the problem of autocorrelation is controlled. The result summary is given in Table 4.

The results of this study indicate that the coefficient of lag debt to GDP ratio is positive and statistically significant. In this stipulation the magnitude of lag debt to GDP ratio is small which shows the existence of less sustainability and confirms that sustainability is weak when public debt is high. If public debt is high with more expenditure and less revenues, there will be increase in interest rate which slows down the growth rate of the economy. The coefficient results of lag debt to GDP ratio is 0.086 showing that 1 percent rise in the debt to GDP ratio will contribute 8 percent increases to the ratio of primary balance to GDP at time t. This shows the long run multiplier effect of debt. An increase in government spending leads to an increase in higher demand for government debt in long run. The result of output gap variable is positive but statistically insignificant. The positive value of output gap indicates that government fiscal policies are countercyclical and indicates crowding out effect in short run. The positive value of output gap suggests that 1 percent increase in output gap will lead to increase primary balance by 1 percent but it is statistically insignificant. The insignificant results of the study support the stance of Khalid et. al. (2007) that on average the sustainability objectives are not considered when making policies in Pakistan.

Table:4 Estimates of the Fiscal Reaction Function, (1984-85 to 2016-17)

Dependent variable is primary balance as ratio of (GDP)

Variable	Coefficient	t-value
Lagged debt	0.0865***	2.7166
	(0.0318)	
Output gap	0.0011	0.7665
	(0.0014)	
Inflation	0.0001**	1.5559
	(8.54)	
PB- R (-1)	0.003**	1.8487
	(0.1566)	
PB- R (-2)	0.4268*	2.7248
	(0.0211)	
Constant	-0.0698	-3.3210
	(0.0201)	

*NOTE: Standard errors in parenthesis. *, ** & *** indicate significance at 10%, at 5%*

The result of the rate of inflation indicates that the coefficient is positive and significant suggesting that economy on stable inflation carries greater primary balances on average, implying that economy needs to carry larger primary balances to support a given position of debt as their inflation level is stable. The positive result of output gap and inflation rate predicts pro-cyclical response of fiscal authority in Pakistan. The pro-cyclical behavior, the fiscal policy reflects that the stabilization objective is not followed while making policies. It negates the Keynesian stance of automatic stabilization of the fiscal instruments as it is not applicable for Pakistan. To meet pro-cyclical response of fiscal behavior the policy makers make contractionary policies which directly target inflation to keep it in limit while leaving aside the output target which leads to decrease in private investment and create uncertainty in the economy. Hence, fiscal policy is ineffective as we described above as well.

5.2. VAR Model

The variable of VAR model includes the steady values of the real GDP growth rate, Pakistan call money rate as real domestic interest rate and U.S. call money rate as real foreign interest rate. The real exchange rate is using quarterly data over the period 1984 (Q_1) to 2017 (Q_2) in Pakistan. The VAR models restricted to 4 lag length criteria selected based on VAR lag order selection criteria. The VAR model is used here to analyze the role of risk factors for debt sustainability. The estimates of unrestricted VAR model are presented in appendix Table A2 and the variance co-variance matrix presented in Table 5.

In this study, we do not discuss the underlying structural shocks; instead, we empirically obtain the covariance of shocks to captures some relationships of these variables. Table 5 is symmetric variance co-variance matrix of residuals obtained from unrestricted vector auto regression model. The diagonal values are the variances and off diagonal value is covariance of shocks of relevant variables. For instance, the U.S call money rate is considered through its covariance with interest rates of Pakistan, exchange rate and GDP growth.

Table: 5 VAR Model [1984 (Q_1) to 2017 (Q_2)]

Variance Co-variance Matrix of shocks				
	Rtus	*Rdt*	gr_t	er_t
Rtus	0.1669	-0.0264	0.0156	0.0181
Rdt	-0.0264	0.0360	-0.0363	0.0104
gr_t	0.0156	-0.0363	0.5090	-0.0397
er_t	0.0018	0.0104	-0.0039	0.86370

It is clear from Table 5, that the volatility of real GDP growth rate shocks is high due to reasonably volatile series. The relationship of U.S call money rate shocks is positive with shocks of real GDP growth rate, exchange rate and negative with the Pakistan interest rate, the negative sign indicates foreign investment increases leads to decrease in the money demand which further decreases domestic interest rate. The negative relationship of exchange rate and real GDP growth rate shocks indicate that lower growth rate leads to depreciate currency of country.

5.3. Risks to Debt Sustainability Analysis in Pakistan

Finally, we examine the sensitivity of the risk analysis through fan charts and a summary indicator of debt sustainability.

5.3.1. Fan charts

Fan charts sum up the risk analysis obtained by frequency distribution of large debt series of stochastic simulations. Different colors define deciles of the debt ratios in distributions. The area in dark blue represents 20 percent confidence interval around the median prediction and the overall color cone shows the confidence interval of 80 percent. The fan charts for public debt-to-GDP ratios of constant policy scenario for Pakistan (2017-2026) is presented in Figure 1 and predicted policy scenario for (2017-2026) period is presented in Figure 2.

Figure 1: Fan Charts for Public Debt-to-GDP Ratios in Pakistan's Economy (2017-2026) Constant policy

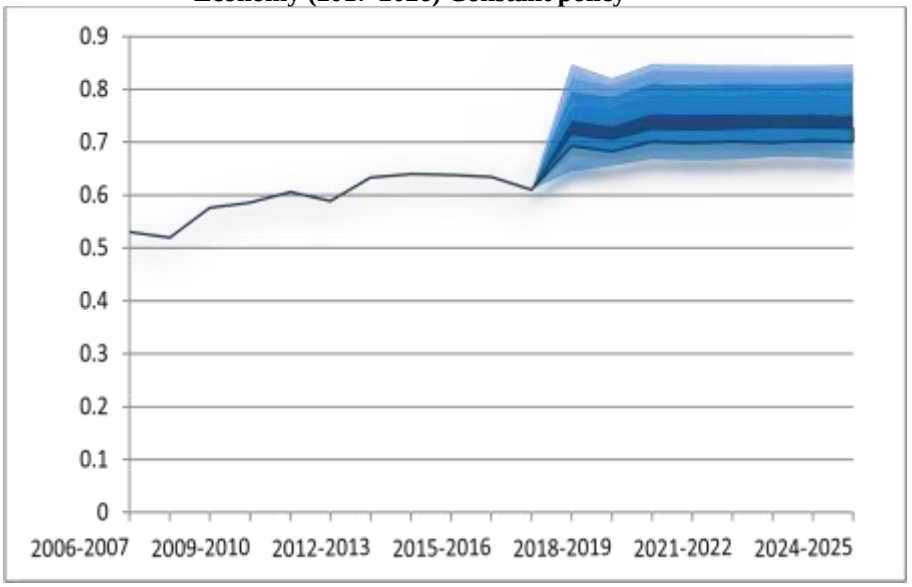
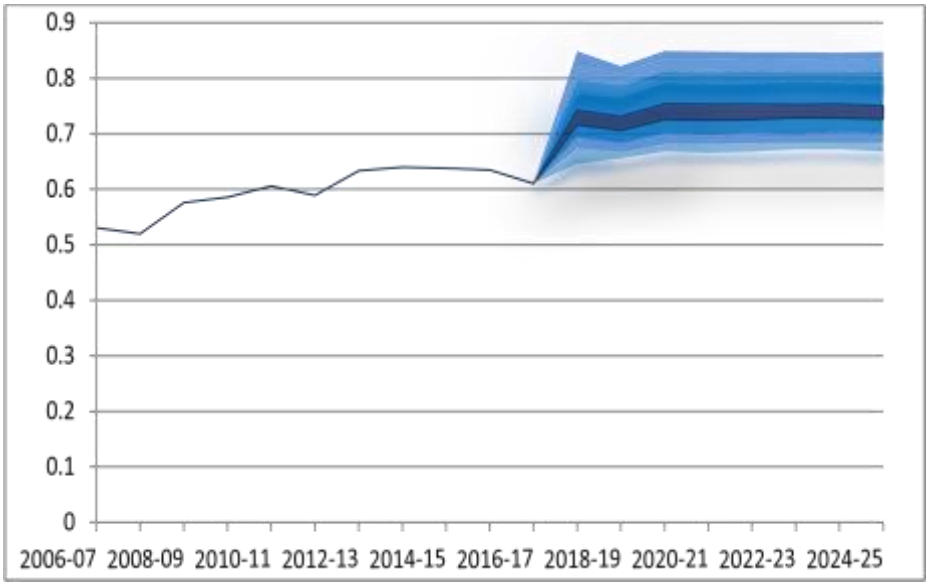


Figure 2: Fan Charts for Public Debt-to-GDP Ratios in Pakistan's Economy (2017-2025) *predicted policy*



Fan Charts present the risk analysis obtained by frequency distribution of large debt series of stochastic simulations both constant and predicted policy for 2017 to 2026 indicating that high primary balance leads to increase debt and high magnitude of upside risks to debt dynamics. The trend shows small magnitude of risk with original debt series but when we incorporate the estimated debt series data, the area of frequency band gets wider which indicates high magnitude of risk. In addition, the result of the fiscal reaction function usually seems more reactive to the public debt and supports that median path of debt is not sustainable in Pakistan (mean that debt ratio is unstable or increasing during the simulation period).The result of fan charts for debt ratios by predicted policy scenario and constant policy scenario is almost similar, as its difference is not applicable in case of Pakistan.

5.3.2. Sustainability Assessments

To check overall risk analysis, this study takes the sustainability indicator which gathered probabilities of debt ratio which will not increase beyond the tolerable amount over same period. Table 6 presents the results of sustainability indicator and shows whether the value of index is of concern or not. Moreover, measuring risk to debt sustainability by combining the probability of decline with contained upside risks are presented in Table A1 in appendix**.** Like a policy maker, we select a threshold level that is 0.6 (i.e., 60 percent of debt to GDP ratio) in this study. All the values of index are below from this threshold level which shows debt series is at risk and appears to be a matter of concern.

Table: 6 Probabilistic Debt Sustainability Assessment

	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+7
Debt ratio lower than in t								
Pak(t=2017)	0.023	0.021	0.007	0.008	0.009	0.009	0.006	0.007
Debt ratio more than 10 percent than in t								
Pak(t=2017)	0.436	0.476	0.398	0.407	0.398	0.389	0.391	0.398
Sustainability index								
Pak(t=2017)	0.01002	0.009996	0.002786	0.003256	0.003582	0.003501	0.002346	0.002786

The sustainability index is defined as [Pr (debt<2016 level)(1-pr (debt> (2016 level+ 10 percent)]*

6. CONCLUSION AND POLICY IMPLICATIONS

This paper has developed stochastic simulation algorithm for obtaining an explicit risk analysis applied to the economy of Pakistan. Overall, we used probabilistic approach to investigate debt sustainability analysis by incorporating endogenous response of fiscal policy which is more realistic assessment of risks. The simulation algorithm is used to generate fan charts and numerical sustainability indicators present an equally supportive visual impression that the future debt level of Pakistan is in crises. The result of Fan charts also confirmed uncertainty and high magnitude of risks towards debt sustainability. Thus, we finally conclude that debt level is not sustainable in Pakistan. There is always ray of hope in the form of policy implications. The probabilistic assessment of debt sustainability can be useful for policymakers in many ways. Firstly, the DSA analysis allows policymakers to have clear picture of risks involved in delaying fiscal adjustment and sustainability of debt can be achieved through fiscal policy reforms. Secondly, the risk profile of debt clearly provides awareness of the risks to public debt for policy makers. This successively helps policy makers to improve the budgetary planning of a country. To promote sustainability, it is suggested that fiscal policies may be designed to control primary balance, inflation rate and interest rate which is helpful to reduce the dependence on public debt and helps to achieve sustainability. As an extension of this work, the cross sectional debt sustainability analysis of Pakistan in relative to other regional countries can be taken. Such analysis can also help in adoption of appropriate policies for better management of debt sustainability.

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Appendix

Table. A1 Measuring Risk to debt Sustainability by Combining the Probability of Decline with Contained

Upside Risks (Probability of decline in column and absence of upside risk in row 1/)

1	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1.00
0.95	0.0475	0.095	0.1425	0.19	0.2375	0.285	0.3325	0.38	0.4275	0.475	0.5225	0.57	0.6175	0.665	0.7125	0.76	0.8075	0.855		
0.9	0.045	0.09	0.135	0.18	0.225	0.27	0.315	0.36	0.405	0.45	0.495	0.54	0.585	0.63	0.675	0.72	0.765			
0.85	0.0425	0.085	0.1275	0.17	0.2125	0.255	0.2975	0.34	0.3825	0.425	0.4675	0.51	0.5525	0.595	0.6375	0.68				
0.8	0.04	0.08	0.12	0.16	0.2	0.24	0.28	0.32	0.36	0.4	0.44	0.48	0.52	0.56	0.6					
0.75	0.0375	0.075	0.1125	0.15	0.1875	0.225	0.2625	0.3	0.3375	0.375	0.4125	0.45	0.4875	0.525						
0.7	0.035	0.07	0.105	0.14	0.175	0.21	0.245	0.28	0.315	0.35	0.385	0.42	0.455							
0.65	0.0325	0.065	0.0975	0.13	0.1625	0.195	0.2275	0.26	0.2925	0.325	0.3575	0.39								
0.6	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.3	0.33									
0.55	0.0275	0.055	0.0825	0.11	0.1375	0.165	0.1925	0.22	0.2475	0.275										
0.5	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225											
0.45	0.0225	0.045	0.0675	0.09	0.1125	0.135	0.1575	0.18												
0.4	0.02	0.04	0.06	0.08	0.1	0.12	0.14													
0.35	0.0175	0.035	0.0525	0.07	0.0875	0.105														
0.3	0.015	0.03	0.045	0.06	0.075															
0.25	0.0125	0.025	0.0375	0.05																
0.2	0.01	0.02	0.03																	
0.15	0.0075	0.015																		
0.1	0.005																			
0.05																				
0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1.00

Upside risk is measured by the probability that the debt-to-GDP into increases by less than a certain amount

Table. A2 VAR Models and Calibration (1984-2016)

VAR coefficient

	Foreign Interest Rate	Domestic Interest Rate	Growth	REEF
Foreign Interest Rate (-1)	0.877024 (9.32370)	0.153745 (1.11378)	0.070987 (0.43225)	-0.08951 (-0.41841)
Domestic Interest Rate (-1)	0.072953 [1.13480]	0.610612 [6.47244]	0.059276 [0.52812]	0.178647 [1.22186]
Growth (-1)	0.023297 [-0.04813]	-0.08775 [-1.24237]	-0.59242 [-7.05016]	0.026107 [-0.10946]
Reef (-1)	-0.0512 [-0.12461]	-0.025987 [-0.43128]	-0.03059 [-0.42670]	1.02041 [10.9267]
Constant	-0.0265 [-0.30073]	0.022461 [1.73812]	0.068327 [4.4424]	0.064651 [3.22813]
R-Square	0.9449	0.6019	0.9786	0.9822
Adj. R-Square	0.9369	0.5445	0.9755	0.9796

Residual Correlation Matrix

	Foreign Interest Rate	Domestic Interest Rate	Growth	REEF
Foreign Interest Rate	1.0000	-0.1078	0.0536	0.0048
Domestic Interest Rate	-0.1077	1.0000	-0.0849	0.0188
Growth	0.0536	-0.0849	1.0000	-0.0599
REEF	0.0048	0.0188	-0.0599	1.0000