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AN ARIMA-ANN HYBRID MODEL FOR MONTHLY GOLD PRICE FORECASTING: EMPIRICAL EVIDENCE FROM PAKISTAN

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ABSTRACT

In this article, we combine linear and non-linear models such as Auto-regressive integrated moving average (ARIMA) and artificial neural network (ANN) to tackle a single model's limitation. The combined approach refers to the ARIMA-ANN model. For empirical analysis, we use gold prices data for Pakistan from 1/7/2003 to 1/6/2021. Firstly, we divided the data into two parts; using the first part, we estimated the models, and the second part is used for models' assessment. We use two error metrics for models' evaluation: Root Mean Square Error (RMSE) and Mean Absolute Error (MAE). We found that ANN performs superior to ARIMA model in terms of forecasting accuracy as shown by RMSE and MAE. A better forecast often leads to improved policymaking. The policymakers and practitioners may adopt the hybrid model for gold prices forecast.
Keywords: Auto-regressive integrated moving average, Artificial neural network, Forecasting, Hybridization, ARIMA-ANN

1. Introduction

Time series forecasting is an important domain of research that has attracted considerable attention from research communities in various practical fields, including economics, medical, finance, business, earthquake prediction, weather forecasting, etc. During the last few decades, many attempts have been made by researchers for the development of efficient prediction models to continuously improve forecasting accuracy (Gooijer and Hyndman, 2006).
The field of forecasting is very dynamic and in the process of continuous improvement/correction. James et al., (2013) has stated that there is no free lunch in statistics. It reveals that no such method exists in the literature that beats all other methods in forecasting under every type of data distribution. Many methods are available in the literature, but unfortunately, still 'no such method exists. In other words, a substantial amount of literature ends up inconclusive. A significant improvement in the existing literature can only be achieved by proposing an ultimate forecasting model, which unfortunately does not exist. Hence, we can conclude that every method performs better in particular situations (depending on data distribution). Therefore, it is necessary to identify the data generating process (DGP) before applying any forecasting tool. Once if discovered that the association between the variables is linear, then linear models usually perform better. In the case of non-linearity, non-linear and non-parametric methods perform better. Although if the autoregressive conditional heteroscedasticity (ARCH) effect is present in

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the data, then the 'ARCH' family of models is good. Following this strategy any such study can be made conclusive.

Prediction has been the area of interest in time series analysis since long. The conventional time series forecasting methods, such as autoregressive, autoregressive moving average, vector auto-regression, and several other models, assume that the data under consideration is generated from a linear process. Linear models are dominant because they can be understood, explored thoroughly and efficiently implemented with meaningful interpretation. Albeit, they may be completely ill-suited if the data at hand has a non-linear structure. It is unreasonable to assume a priori that a linear process generates a certain sample. Real-world phenomena are usually non-linear (Granger and Terasvirta, 1993). Over past several decades, various non-linear time series models such as the bilinear model (Granger and Anderson, 1978), the autoregressive threshold model (Tong and Lim, 1980), and the autoregressive conditionally heteroscedastic model (Engle, 1982) have been developed. Although, these non-linear models are still limited as an explicit nexus for the underlying data series must be hypothesized with a little understanding of the underlying mechanism. In reality, formulating a non-linear model to a specific data set is challenging since too many possible non-linear patterns exist. A pre-specified non-linear model may not be typically sufficient to occupy all the important features (Zhang, 2001). The alternative methods that overcome such problems fall under machine learning (ML) algorithms known as Artificial neural networks (ANN).

Artificial neural network (ANN) is a famous and well-accepted technique due to its flexibility to non-linear data sets and reliable results for financial time series. It may be unsuitable for approximating pure linear and/or non-linear models to complicated non-linear phenomena and dealing with linear and non-linear correlation structures. The use of hybrid or combined models has become a widespread practice for overcoming the limitations of model components and raising accuracy to the highest standard (Khashei and Bijari, 2011).

Our study goal is to make a hybrid of both linear (ARIMA) and non-linear (ANN) models to enhance the forecasting accuracy for empirical illustrations. Hence, our study considers the gold prices data to compare the forecasting ability of individual models with a hybrid model (ARIMA-ANN).

In forecasting, it is needed to improve the forecast accuracy up to the maximum level. More specifically, sound macroeconomic policies are not possible without empirical analysis and projections of major macroeconomic indicators. Therefore, various univariate and multivariate tools have been developed to handle data-noise and to improve forecast accuracy. However, it is a fact that the real-world phenomenon is neither purely linear nor non-linear, and thus linear and non-linear models often fail to capture the trend in the data. This work combines linear and non-linear models to establish a hybrid, i.e., ARIMA-ANN, that captures the linear and non-linear parts of a series, improves the predictive capability compared to the individual linear (ARIMA) and non-linear models (ANN).

Up to our knowledge. none of the articles have considered hybrid models using gold prices data in Pakistan context. Our study focuses on gold prices data. Gold is a precious metal, also used as investment too and is different from other metals and assets. It is highly liquid and sensitive to price changes. Most of the gold is bought in the form of jewelry items.

The remaining paper is arranged in following sections. Section 2 covers literature. Section 3 is about data and methodology. Section 4 discusses the empirical results. Finally, section 5 concludes.

2. Literature Review

Over the years, a long list of time series models has been established for forecasting in literature. The autoregressive (AR), moving average (MA) and ARIMA are some well-known statistical predictive models that predict time-series observations based on certain linear functioning of previous values and white noise components (Box and Jenkins, 1976; Zhang, 2003). These models thus impose an intrinsic restriction of linearity on the data generation process. To address this, literature has also devised many non-linear models. ANN having several outstanding characteristics, is amongst the most popular (Zhang et al. 1998; Hamzaçebi, 2008). To rationally improve forecasting accuracy, Zhang (2003) has integrated both ARIMA and ANN models. Meyler et al. (1998) have predicted the Irish inflation by applying the ARIMA model and argued that ARIMA models are theoretically more validated and can be unexpectedly robust comparative to rival models. Tkacz and Hu (1999) has argued that the ANN algorithm is more robust than conventional approaches. These models can capture more essential non-linearities in the underlying data.

Khani et al. (2021) have evaluated the performance of Convolutional Neural Networks (CNN) long short-term memory (LSTM), vector sequence output LSTM, Bidirectional LSTM, and encoder-decoder LSTM considering gold prices data. They have inferred that vector sequence output LSTM is superior to the other three models. Chai et al. (2021) have performed STL-ETS, ANN, and Bayesian structural time series models to predict gold return and compared them with the benchmark model. They concluded that STL-ETS fits the data noisy very well and enhances the prediction accuracy comparatively. Bakar et al. (2021) have concluded that the support vector machine and ARIMA hybrid are more predictive than ARIMA.

Ghashami et al. (2021) have combined ANN and Particle Swarm Optimization to forecast the stock market index i.e., NASDAQ. Mehtab and Sen (2020) have predicted the stock market by using machine learning and deep learning techniques. Similarly, Patel et al. (2015b) have recommended a dual-stage fusion procedure to forecast the stock market index. The Support Vector Regression (SVR) model is used in the initial phase, followed by ANN, Random Forest (RF), and SVR in the second stage. The final model results in SVR-ANN, SVR-RF and SVR-SVR.

Similarly, Saeed et al. (2000) have used the ARIMA model to forecast wheat production in case of Pakistan. Ning et al. (2010) have used the ARIMA methodology to model economic growth. The study restricted itself to non-seasonal time series data and attempted to forecast the GDP of Shaanxi province in the succeeding six years based on the ARIMA model using time series data spanning 1952-2007. Sami et al. (2012) have assessed the rate of dust fall utilizing the ARIMA model with other models in Quetta, Pakistan. Farooqi (2014) has carried out an ARIMA model to predict the future annual values of imports and exports in the case of Pakistan.

Moshiri and Cameron (2000) have performed the ANN and other traditional statistical methods to forecast Canadian inflation. They argued that the ANN algorithm could resolve complex problems, and, unlike traditional models, it does not require the assumption of linearity. Burney et al. (2005) have forecasted Karachi stock exchange shares by using ANN. Haider and Hanif (2009) have forecasted inflation based on monthly data via ANN and simply compared it with ARIMA. They showed that ANN is superior to ARIMA in forecasting. Awan et al. (2012) have proposed a hybrid non-linear Autoregressive exogenous model combined with feed Forward Network. They have forecasted the long-term industrial load and showed that the proposed hybrid model NARX based FFNN is more robust in forecasting.

In the context of Pakistan, many previous studies have adopted AR, ARIMA, ANN, VAR, etc, for forecasting purposes, but there is a lack of hybrid models. As elaborated in section 1, those hybrid models are more suitable for forecasting by dint of capturing linear and the non-linear trend in the data and, consequently, providing an accurate forecast. Our study focuses on hybrid ARIMA-ANN and compares it with the separate ARIMA and ANN using gold prices data.

3. Data and Methodology

The following section provides the detailed account for the data and methodology.

3.1. Data

This study uses gold prices data per ounce (in the dollar) spanning from 1/7/2003 to 1/6/2021. The source of data is Yahoo finance website. However, our study compares the hybrid model's predictive ability with ARIMA and ANN using gold data. It is a fact that comparisons among several models are often made on test data. Therefore, we divided the data into two parts i.e., training data (1/7/2003 to 1/7/2018) and testing data (1/8/2018 to 1/6/2021). We train the model on the training set, and then evaluate using the testing set, we evaluate the performance of the underlying models.

3.2. Methodology

In this section, we discuss a number of Linear and Non-Linear models including ARIMA, ANN, and Hybrid model.

3.2.1. Linear and Non-Linear models

Time series prediction is a rapidly developing field and offers numerous opportunities for future development. One is to concatenate some techniques, that is, to enhance the accuracy of the prediction. Much work has been done in this respect, and numerous combinations of techniques have been recommended in literature (Armstrong, 2001; Zhang, 2003; Armstrong, 2006).

ARIMA: The ARIMA is a prominent statistical model for stationary and non-stationary time series forecasting in the last few decades. This model often includes autoregressive (AR) and MA models, incorporating a phrase for data transformation known as differentiation. However, the ARIMA model is limited by the linearity assumption that is difficult to completely meet in actual-world applications or by utilizing solely historical information as inputs. The formulation of the ARIMA model comes up with an auto-regressive moving average model, which is a special case of the ARIMA model, given in Equ. (1).

$$Y_t = c + \sum_{i=1}^{m} \alpha_i \, Y_{t-i} + \varepsilon_t - \sum_{j=1}^{n} \beta_j \, \varepsilon_{t-j} \tag{1}$$

The ARMA model forecast one step ahead (Y_t) by utilizing past values of time series (Y_{t-1}, Y_{t-2}, …, Y_{t-m}) and past residuals (ε_{t-1}, ε_{t-2}, …, ε_{t-n}), where α_i and β_j are the unknown parameters; c is an intercept term; ε_t is independently and identically distributed with mean zero and variance δ^2; m and n are the orders of past values to be incorporated in the model.

To simplify the above formula, backward shift operator (A), defined as $A^i Y_t = Y_{t-i}$, is replaced for an ordinary algebraic symbol in Equ. (1). Hence, the ARMA model can be formulated as follows:

$$Y_t = c + \sum_{i=1}^{m} \alpha_i Y_t A^i + \varepsilon_t - \sum_{j=1}^{n} \beta_j \varepsilon_t A^j \quad (2)$$

Then, post adjustment, the terms related to Y_t in (2), we get the ARMA model as below:

$$(1 - \sum_{i=1}^{m} \alpha_i A^i) Y_t = c + \left(1 - \sum_{j=1}^{n} \beta_j A^j\right) \varepsilon_t \quad (3)$$

To rewrite the expression in a compact form,

$$\alpha_m(A) Y_t = c + \beta_n(A) \varepsilon_t \quad (4)$$

Where

$$\alpha_m(A) = 1 - \sum_{i=1}^{m} \alpha_i A^i \quad (5)$$

$$\beta_n(A) = 1 - \sum_{j=1}^{n} \beta_j A^j$$

are 'so called' AR operator $\alpha_m(A)$ and the MA operator is $\beta_n(A)$, respectively.

Although the ARMA model fails to adjust the unit root effect in time series data, difference transformation is needed to achieve stationarity and obtain reliable results. In the following way, the integration term is adjusted.

$$\alpha_m(A)(1 - A)^s Y_t = c + \beta_n(A) \varepsilon_t \quad (6)$$

Artificial neural networks (ANNs) approach: The potential number of non-linear structures which may be utilized to describe and predict a time series is vast when the linear constraint of the model form is relaxed. A better non-linear model should be "general enough to capture the certain non-linear structure of the data" (Gooijer and Kumar, 1992). ANNs are part of such models which can approximate non-linearities in the data in an effective way.

ANNs are versatile computer frameworks to simulate a wide variety of non-nonlinear issues. One of the main advantages of the ANN models over other non-linear models is that the ANNs can approximate huge class of functions with high precision (Khashei and Bijari, 2011). Its power is derived from the concurrent processing of data. During the model construction, no prior assumption about the model shape is needed. The network model is, instead, mainly defined by the data features.

One of the most used functional forms for time series prediction is a single hidden layer feedforward network (Zhang et al. 1998). Cyclical connections define a matrix of three layers of basic processing units. Following is the mathematical illustration of the association between the output (M_k) and the inputs (M_{k-1}; M_{k-2}, …, M_{k-p}).

$$M_k = \alpha_0 + \sum_{l=1}^{q} \alpha_l g\left(\gamma_{0l} + \sum_{i=1}^{p} \gamma_{il} M_{k-i}\right) + e_k \quad (7)$$

The model parameters, often known as connection weights, are α_l *(l = 0, 1, 2, …, q) and* γ_{il} *(i = 0, 1, 2,… p; l = 0, 1, 2,…., q)*; *p* is the number of input nodes, and *q* is the number of hidden nodes. As a hidden layer transfer function, the logistic function is frequently employed and can be written as:

$$g(x) = \frac{1}{1+exp(-x)} \tag{8}$$

Hence, the ANN model of (8), performs a non-linear functional mapping from the past observations (M_{k-1}; M_{k-2}, …, M_{k-p}) to the future value M_k, i.e.,

$$M_k = f\,(M_{k-1}, M_{k-2}, \dots, M_{k-p}, w) + e_k \tag{9}$$

Here w is a vector of all parameters, and f is a function ascertained via network structure and connection weight. The neural network, therefore, corresponds to an autoregressive non-linear model. Equation (9) entails one output note in the output layer and yields one step ahead forecast.

Simple ANNs algorithms are compelling in prediction. Neural networks with one or two hidden nodes are often better forecast for time series data (Zhang et al. 1998). The ANN method consists of four sections: (i) features selection, (ii) training and testing formulations, (iii) architecture, and (iv) forecasting.

Hybrid model: In short, establishing a hybrid model requires two stages. In the first stage, the linear component of the data is analyzed using an ARIMA model. We build a neural network using the residuals extracted from the estimated ARIMA model in the second stage. As the ARIMA model cannot capture the non-linear pattern in the data, the residuals of the ARIMA model hold some important information regarding non-linearities. The output from the ANNs can be utilized as forecasts of the residuals for the ARIMA model. In discovering different structures, the hybrid model uses ANN and ARIMA model's unique characteristics and strengths. Through different models, linear and non-linear patterns may be adequately modeled and merge their predictions to enhance the overall modeling and prediction performance (Zhang, 2003). Figure 1. portrays the entire scenario.

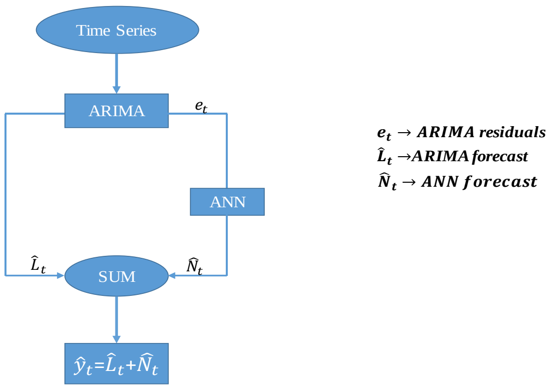

Figure 1: ARIMA-ANN

4. Empirical results

We now consider the data for gold prices and conduct the analysis. In Figure 2, we can observe that the series at level increases over time, which reflects the unit root problem. In other words, the statistical properties such as mean, variance, and covariance of the original series are not constant over time. To remove this pattern from the data, we take the natural logarithm of the underlying series and apply difference transformation. Graph of the first differenced series is shown in Figure 3. From the graph of the first difference, we can conclude that the series are difference stationary. In the first row of Figure 4, the ACF plot is gradually declined is another evidence of non-stationarity. However, after transformation, the ACF plot in the second row of Fig. 4 decreases rapidly. Hence, we can use the stationary series for further modeling and forecasting. A certain pattern in the ACF and PACF plots is associated with a certain order of q and p, respectively.

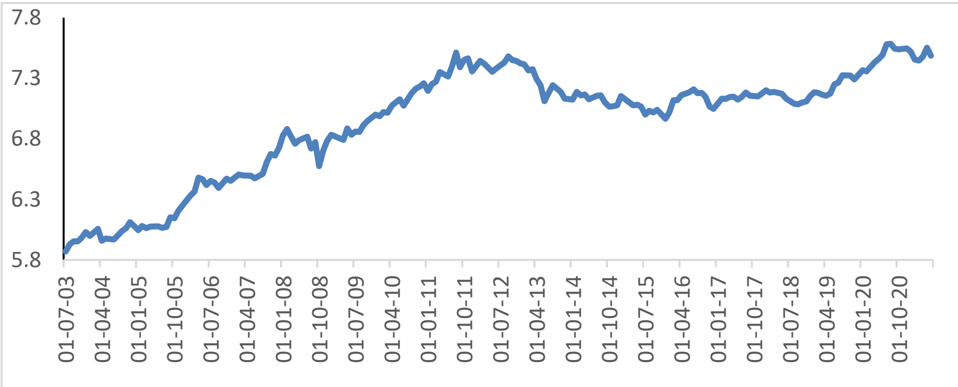

Figure 2: Gold prices at level

For robustness, we use the standard unit root test known as the Augmented Dickey-Fuller test (ADF). Using the ADF test, it is assumed that the variable contains a unit root under the null hypothesis. The result of the unit root test is presented in Table 1. The result reveals that gold prices are non-stationary at a level. As we take the first difference of the series, it turns out to be stationary.

Table 1. Unit root testing

	At level	First difference	Conclusion
Variable	Constant with trend		
Gold Prices	-2.825	-5.963*	I(1)

Note: * shows significance at 5 percent.

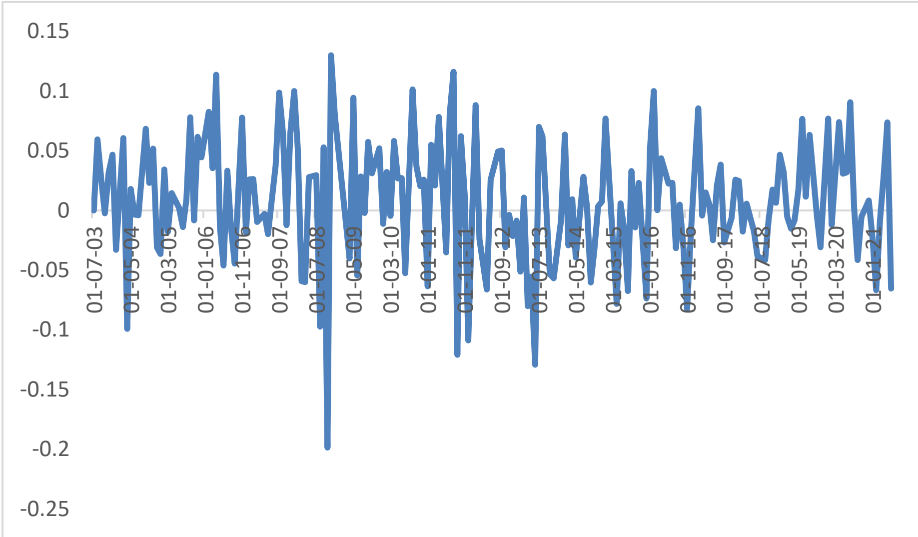

Figure 3: First difference of gold prices

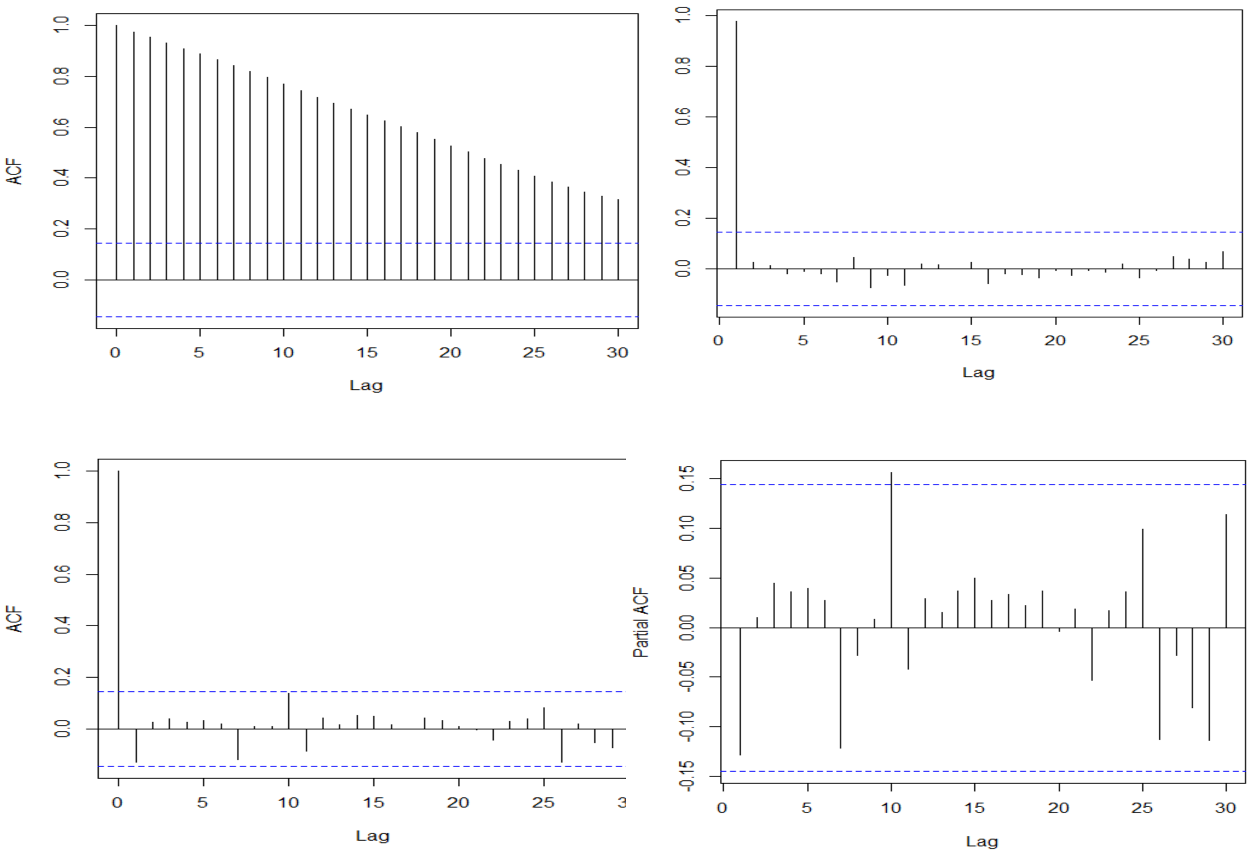

Figure 4: ACF and PACF for level (**first row**) and differenced data (**second row**)

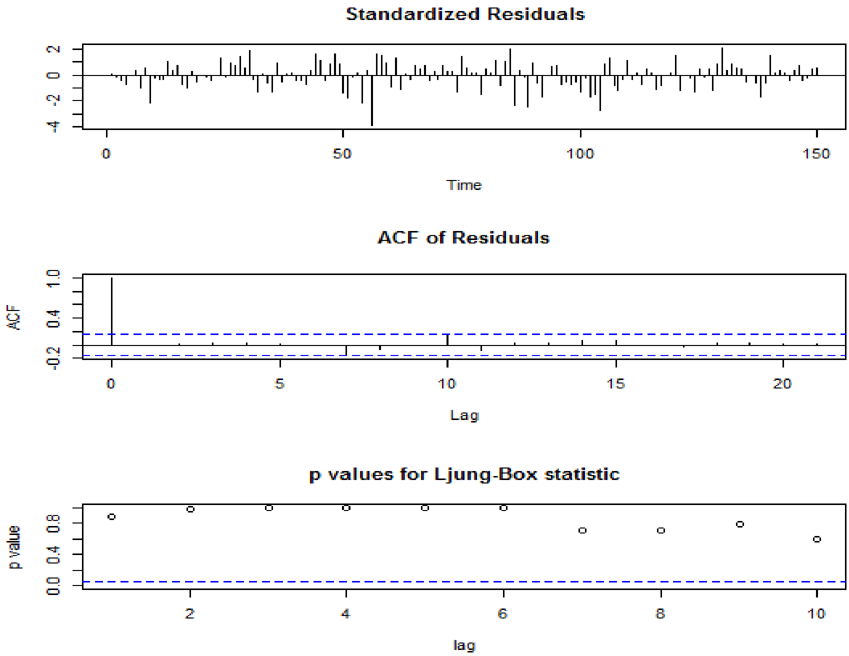

Figure 5: Diagnostic check

One of the computing techniques, or artificial intelligence, has been extensively studied and used in time series forecasting, known as Artificial neural network (ANN). Post fitting the ARIMA model, now we use the ANN algorithm for the prediction of gold prices. It is often used for short-term prediction. In the ANN algorithm, 1 input and 2 hidden layers are selected with 1 output layer of neurons using the trial-and-error method. Finally, we make the hybrid of both, i.e. ARIMA and ANN, to obtain more accurate forecasts.

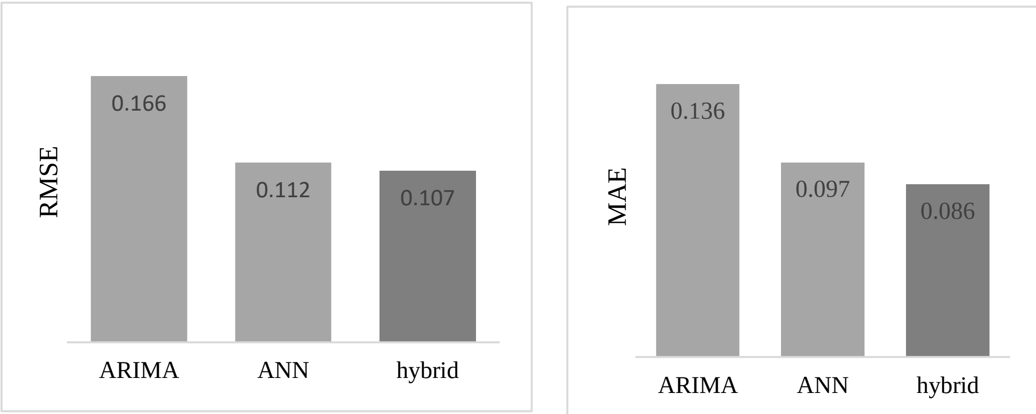

Figure 6: Forecast comparison across univariate models

One way to know the randomness of the residuals of the fitted model is by considering the graphs of the residuals and applying Box-Ljung test. The residuals plot of the fitted model is displayed in Figure 5. There is no significant spike in autocorrelations. In addition, the p-values associated with the Box-Ljung test are also large. Therefore, we can conclude that the residuals are random. In other words, there is no autocorrelation issue in the fitted model. Moreover, we examined the residuals of ANN and hybrid are white noise, and there is no autocorrelation issue.
For model evaluation, two error metrics are utilized: Root Mean Square Error (RMSE) and Mean Absolute Error (MAE). RMSE and MAE are computed in the following way:

$$RMSE = \sqrt{\frac{1}{M}\sum_{c=1}^{M}(Y_c - \hat{Y}_c)^2} \qquad (10)$$

$$MAE = \frac{1}{M}\sum_{c=1}^{M}\left|Y_c - \hat{Y}_c\right| \qquad (11)$$

According to Figure 6, the RMSE and MAE associated with ANN are less than the RMSE and MAE of the ARIMA model, which demonstrates that ANN forecast is closer to the actual values than ARIMA. However, as we combine these two models in the form of ARIMA-ANN; as a result, the corresponding RMSE and MAE are further declined. We can infer that the hybrid model produced the lowest forecast error compared to individual ARIMA and ANN. It illustrates that a combination of linear and non-linear models captures the data pattern more suitably.

5. Conclusion

Improving forecasting accuracy in the time-series domain is an important and often tricky task confronting data analysts in different areas. Despite many time series models are available, the research for boosting the capability of prediction models has never stopped. Some large-scale forecasting competitions with a mass of widely utilized time series forecasting models concluded that merging forecasts from two or more models usually enhances forecast accuracy, mainly when the combined models are different. Hybridization that decomposes a series into two parts, i.e., linear and non-linear form, is one of the most well-known hybrid models and has been shown in the literature to be successful for single models.

This work has combined the linear (ARIMA) and non-linear models (ANN) to tackle a single model's limitation. The combined approach refers to the ARIMA-ANN model. For models' comparison, we collected data of gold prices from 1/7/2003 to 1/6/2021. We estimated the models on the training set and assessed them on the testing set. The findings show that ANN outperforms the ARIMA model in terms of forecasting. As we combined their forecast, the ARIMA-ANN achieved an outstanding forecast compared to ARIMA and ANN. We conclude that ARIMA-ANN produces better forecast than ARIMA and ANN. It is obvious that better forecast leads to improved policy in the future. Hence, we recommend the policymakers and practitioners may adopt the hybrid model for predicting gold prices behavior.

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