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Asymmetric Cointegration and the J-curve: New Evidence from Pakistan and China Commodity level trade balance models

Mehmood Ul Hassan[1], Saqib Ullah Khan[2], Syeda Aqsa Sherazi[3], Abdul Hameed[4]

Abstract

New econometric methods immensely raised the interest in assessing the J-curve and the old theories of economics. Therefore, in this paper, we employ the linear autoregressive distributed lag (ARDL) framework to assess the symmetric effect of exchange rate on commodity level trade balance between Pakistan and China by covering the annual data from 1980-2024. In the next phase, we employ the newly emerged the nonlinear NARDL model to capture the asymmetry in the behavior of the exchange rate. The main feature of the nonlinear ARDL model allows us to divide the real bilateral exchange rate into two parts: depreciation and appreciation. Consequently, we separately assessed depreciation and appreciation impact on the industry-level trade balance. In the case of Pakistan-China industry-level trade, we conclude that the nonlinear model estimation produces stronger arguments in favor of the J curve and short-run real bilateral exchange rate than what has been presented in the linear model.

Keywords Asymmetry; J-curve; Nonlinear ARDL; Real Bilateral Exchange Rate; Commodity Trade

JEL Classification: F31, F32, F41, C32, C51

1. Introduction

Pakistan has quite a longstanding experience of having a large trade imbalance. As like other nations, it really has depended on both currency devaluation and depreciation to enhance its trade balance. By 1952, the Pakistani rupee has been devalued for the first time. Following that, there have been several occasions where the Pakistani rupee has lost value. The most prominent currency devaluations occurred in 1972 and 1996. The rupee's depreciation was anticipated to enhance exports while restricting imports. Nevertheless, it's possible that the lack of improvement in the trade imbalance is related to the inflationary impacts of nominal devaluation. As a result, we must combine the nominal exchange rate and price increases into a single variable and take real exchange rate volatility into account.

The real exchange rate is theoretically an essential determinant of exports and imports because it is the parameter of a country’s global competitiveness. A condition known as Marshal Lerner condition affirms that “devaluation improves trade balance if the sum of import and export elasticity is greater than unity (Rose, 1990). The trade balance initially

[1] Ph.D. Scholar, School of Economics, Quaid-e-Azam University Islamabad. Email (Corresponding Author): Mehmoodulhassan_16@outlook.com
[2] Ph.D. Scholar, Groupe de Recherche Angevin en Économie et Management (GRANEM), University of Angers, France. *Email:* Saqibkhattak99@gmail.com
[3] Quaid-e-Azam University Islamabad. *Email: Syeda.aqsashah123@gmail.com*
4 Lasbela University of Agriculture Water & Marine Sciences Sub-Campus Wadh Khuzdar Balochistan *abdulhameedmengal217@yahoo.com*

worsens following currency depreciation in the short run due to the adjustment lags and contracts but improves in the long run. This is known as J-curve phenomenon (Magee, 1973).

In the real world, there are two separate definitions of J-curve that have been given by economists. The traditional definition states that the trade balance will worsen first and will improve latterly. This indicates that the estimates of the exchange rate would behave as negative at the shorter lag and would be accompanied by a substantial and positive coefficient at the greater lags to produce the J-curve phenomena (Bahmani-Oskooee, Bose, & Zhang, 2019). On the other hand, the revised interpretation of J-curve by Rose and Yellen (1989) is focused with long-run parameters. It shows that in the short run the real exchange rate will be having a negative coefficient followed by a significant positive coefficient in the long run.

The reason for not finding any significant effect is because of aggregation biasness. To make the results reliable and remove the aggregation problem, we disaggregate trade flows between Pakistan and China to the industry level instead of total flows. It enabled us to investigate the impact of movements in exchange rates on the industry-level balance of trade. The second important contribution of the study is the distinction of symmetric studies that used linear models. In this study, we employ the nonlinear model by rejecting the exchange rate symmetry assumption which treats appreciation and depreciation in the same manner. For this purpose, we split the exchange rate into two parts, depreciation, and appreciation. At this stage, we have got an idea that currency appreciations and depreciations affect trade balance in different manners and there is a need to be assessed on a separate basis. However, the symmetry assumption is denied based on the study of (Bussiere, 2013) who suggested that import and export prices react asymmetrically to the movements in the exchange rate.

Moreover, we believe that if the traded goods prices respond to changes in the exchange rate in an asymmetric manner, then it is natural to anticipate that exchange rate movements will affect trade balance in an asymmetric manner. Previous studies like the study of (Aftab & Khan, 2008; Bahmani-Oskooee & Cheema, 2009) by keeping the symmetric assumption did not verify the RER effect on Pakistan trade balance. In these studies, it was found that currency depreciation adversely affects Pakistan's balance of trade. However, there is no evidence that can support the view that rupee appreciation generates a contradictory effect. We doubt that the effects of the exchange rate are asymmetric. For that purpose, the commodity trade of Pakistan and China both in the linear and nonlinear framework is being analyzed in this study. Nevertheless, irrespective of the industries' retort to the dynamics of the exchange rate, it cannot be concluded from the studies of aggregate trade balance behavior. So, disaggregating the trade balance data to commodity level is a need of time, it has benefits like the industries trade balance exploration assist policymakers to understand the extent and direction of each industry's response to exchange rate changes

This study enhances our insight of the dynamic influence of exchange rate changes on Pakistan's Industry-level trade balance. It assists the researchers to know the degree to which real exchange rate shifts should be considered when designing, managing, forecasting, and regulating trade flows between Pakistan and China. Furthermore, whether the exchange rate can be a good indicator to improve the trade balance & give a reference to the central bank in the policy-making decision and for further research in this area. The remainder of the paper is organized in the following way. The next section is the literature review, methodology of the study is given in third section. The fourth section describes the results and discussions of the paper. The final section is all about the conclusion and policy implications of the study.

2. Literature Review

The evidence-based review of the literature on the J curve is split into two categories: symmetric and asymmetric research. In the first part, we present summary of the studies that analyze the j-curve symmetrically. The second part discusses asymmetric investigations that include a variety of research that employ nonlinear models.

Many studies were conducted after these theories' development for both developed and developing nations over the previous 50 years. Nevertheless, still, there is no agreement in the existing empirical literature about the relationship between foreign exchange rates and trade balance in developing countries. In aggregate-level studies, by taking symmetric assumption of the exchange rate, the study of (Miles, 1979; Himarios, 1989; Rose & Yellen, 1989; Bahmani-Oskooee & Alse, 1994; Himarios, 1989; Lal & Lowinger, 2002; Shahzad, Nafees, & Farid, 2017) find no effect in most of the countries. For bilateral studies, (Rose & Yellen, 1989) criticized aggregate level data and prior findings and investigates the J-curve for the American economy. The trade balance was modeled as dependent on domestic income, foreign income, and the real exchange rate. In the same study, the results obtained for aggregate data demonstrated J-curve existence for America while the bilateral level study doesn't show J-curve existence.

In a similar pattern, the study of (Wilson & Tat, 2001; Arora, Bahmani-Oskooee, & Goswami, 2003; Bahmani-Oskooee & Wang, 2006; Narayan, 2006; and Bahmani-Oskooee, Economidou, & Goswami, 2006) find that exchange rate has different effects on trade balance at bilateral level trade. The disaggregate level studies of (Ardalani & Bahmani-Oskooee, 2007) stand as an example where they paid attention using monthly data, the study examined the J-curve across 66 U.S. trading industries and identified evidence of the J-curve in six sectors, along with 22 industries a long-run direct relationship. In Pakistan, the exchange rate is always being used as an important factor to enhance exports and reduce imports. For this purpose, (Hameed & Kanwal, 2009; Bahmani-Oskooee & Cheema, 2009; Jalil, Abbasi, & Bibi, 2016; Rehman & Afzal, 2003; Iqbal, Nosheen, Tariq, & Manan, 2015; Bahmani-Oskooee, Iqbal, & Muzammil, 2017) investigated that whether trade balance is affected by exchange rate or not. All these studies concluded in mixed findings.

The research on balance of trade has been established by holding the notion that the rate of exchange would have an asymmetrical influence. For this purpose, many researchers, for instance, (Chinn & Frankel, 1991; Lin & Fu, 2015; Jibrilla Aliyu & Mohammed Tijjani, 2015; Qayyum, Nazir, & Jawad, 2016; Bahmani-Oskooee, Bose, & Zhang, 2017; Bahmani-Oskooee & Fariditavana, 2016; Soleymani, Chua, & Saboori, 2011; Bahmani-Oskooee & Harvey, 2017), have researched and found additional proof that nonlinear models are more in support of the J-curve as compared to the linear models.

Using the ARDL model, Narayan, (2006) revealed co-integration between China and its trade with the United States, and found an improvement in China's trade balance when the currency depreciated. The J-curve, on the other hand, was not discovered. In a similar study Bahmani-Oskooee, Economidou, & Goswami, (2006), looked at the J curve between China and its 13 trade partners and found it only with Hong Kong and the United Kingdom.

Using data from 177 sectors, Bahmani-Oskooee & Kovyryalova, (2008) discovered a significant short-run relationship between exchange rate and trade balance in 60% of industries, but no clear shape of the J curve. According to its revised definition, the J curve was seen in a variety of sectors. In a similar research Bahmani-Oskooee & Mitra, (2008), data for 38 sectors in India's trade balance with the US was used to examine the effect of real depreciation, and J curves were discovered for three industries using the old and eight industries using the new definition of J curve. Using the ARDL and ECM models, Soleymani, Chua, & Saboori, (2011) investigated the short-run impact of depreciation on the trade balance of Malaysian trade with Japan using 67 industries. Their empirical findings support J curve only for 22 sectors. In the short term, Korean exports and imports were shown to be more responsive than Japanese exports and imports, but less so in the long run.

Bahmani-Oskooee and Harvey, (2017) studied the trade balance of Malaysia and her top trading partners at the bilateral level. Like the previous studies, he came up with the view of asymmetry in the manner of the exchange rate effect on the trade balance. Nonlinear ARDL model is used to check the short and long-run effects of currency appreciations and depreciation. The study found adjustment asymmetry, short and long-run impact asymmetry in the bilateral trade between Malaysia and Asian countries. After the development of ECM and Cointegration, the j-curve as defined as short-run deterioration and long-run improvements has changed. the standard models like the ARDL approach of Pesaran, Shin, & Smith, (2001) suppose the relationship between economic variable as linear. But the study of Bahmani-Oskooee & Fariditavana, (2016) on the bilateral trade of the United States and its top selected trading partners assumed that the relationship is nonlinear in nature. The study finds more evidence in support of the J curve as compared to the linear models.

Overview of the Literature

The literature on the J curve consists of two types of studies, i.e., Symmetric, and Asymmetric studies. The symmetric studies in the literature have employed aggregate trade data, bilateral trade data and few have used the commodity/industry level data. Their findings, however, are mixed and ambiguous and don’t give a specific pattern of the J curve. The second type of studies in the literature believes that the relationship between the exchange rate and the trade balance is nonlinear. In case of Pakistan the earlier studies have either used aggregate trade data or have employed bilateral trade data emphasizing on the symmetry assumption, but no one has attached importance to the view that the relationship can be nonlinear at commodity /industry level. So, in this study, we are disaggregating Pakistan’s trade with China at commodity level and investigating the J curve both in linear and nonlinear framework.

3. Methodology

To investigate the asymmetric Cointegration and the J-curve based on commodity level trade balance models between Pakistan and China, Researchers collected the data from secondary sources: the World Development Indicator (WDI) as well as the World Integrated Trade Solutions (WITS). The time period was chosen depending on the availability and accessibility of the data, which ranged from 1980 to 2024. The data for the nominal exchange rate, and real gross domestic product for Pakistan and its trading partner China was taken from WDI, whereas the data on commodity trade flows were taken from the WITS.

Before proceeding to ARDL and NARDL, it is necessary to see that whether the data is stationary or not. for that purpose, we used Augmented Dickey and Fuller (ADF) unit root test developed by (Dickey & Fuller, 1979). After unit root, this study uses both linear and nonlinear approaches. The linear model of (Rose & Yellen, 1989) and (Bahmani-Oskooee & Fariditavana, 2016) nonlinear model has been followed. The long-run specification of the model is given below:

$$LnTB_{i,t} = a + bLnY_{PAK,t} + cLnY_{Chn,t} + dLnREX_t + \epsilon_{t,t} \quad (1)$$

Following the work of Bahmani-Oskooee and Hajilee (2009) and Bahmani-Oskooee and Harvey (2010), we use Trade balance (TB_i) defined as the ratio of the dollar value of Pakistan export of i_{th} commodity to China and the dollar value of Pakistan's imports from China of i_{th} commodity ($^X/_M$). This method has two advantages. Firstly, it is unit-free. Secondly, this method has no issue of taking logs as the trade balance is in ratio form rather than in absolute values. The present study took 1-digit industries that trade between Pakistan and China. While $Y_{Pak}$and Y_{Chn} has been used as a proxy of income and production capacity of Pakistan and China respectively. The bilateral exchange rate is useful in this research work as it measures the prices of the goods of one country in terms of another country's good price. The bilateral exchange rate is defined as $REX_i = (P_i.{}^{NEX_i}/_{P_{Pak}})$ where NEX_i are the values of nominal exchange rate defined as Number of units of China Yuan per Pakistani Rupee, P_{Chn} is the price level in China (also measured by CPI) and P_{Pak} is the price level in Pakistan (as measured by CPI). Thus, a decline in REX_i reflects a real depreciation of the Rupee. Following Guechari (2012), Mohammad and Hamza (2012), and many other studies have used the real bilateral exchange rate in their study.

The following equation is updated to account for short-run impacts using the Error Correction Framework.

$$\begin{aligned}\Delta LnTB_{i,t} = \alpha + \sum_{j=1}^{n} \beta_j \Delta LnTB_{i,t-j} + \sum_{j=0}^{n} \delta_j \Delta LnY_{PAK,t-j} \\ + \sum_{j=0}^{n} \gamma_j \Delta LnY_{Chn,t-j} + \sum_{j=0}^{n} \pi_j\, \Delta LnREX_{i,t-j} + \lambda_1 LnTB_{i,t-1} + \lambda_2 LnY_{PAK,t-1} \\ + \lambda_3 LnY_{Chn,t-1} + \lambda_4 LnREX_{t-1} + \mu_t \quad (2)\end{aligned}$$

Equation (2) gives an edge versus equation (1) which would be the typical error correction model. This equation empowers us utilize the one-step OLS approach to reflect the association between the variables both in the long and short term. The assessments of short-run consist including first difference sign whereas $\lambda_2, \lambda_{3,} \lambda_4$ are indeed the long-run parameters where it is standardized predictions of λ_1. It is necessary for the coefficients of the long run to be meaningful and true that there must be co-integration amongst the variables. F test is recommended by (Pesaran, Shin, & Smith, 2001) to make a combined significance for the linear combination of 1st lag variables to give evidence of Cointegration and for the large sample size, a set of tabulated critical values is offered. In this paper, we use critical values offered by (Narayan & Narayan, 2005) due to the small sample size. In some cases, when the F test cannot be held and gives inconclusive results, we refer to the ECM version of the ARDL suggested by (Kremers, Ericsson, & Dolado, 1992).

Nonlinear Autoregressive distributed lag model to Cointegration

The main assumption in the linear framework presented in Equations 1 & 2 is that the movements in the exchange rate affect trade balance in an asymmetric manner. To become more specific, the elasticity of currency appreciation and devaluation over trade balance is assumed to be the same. However, based on Shin et al. (2014), we argue that the exchange rate has an unequal effect on industry balance of trade. Researchers use the partial sum technique to differentiate devaluation from appreciation for this objective. Second, we generate to show rupee appreciation and devaluation, with the rate of change in the currency rate.

$$POS_t = \sum_{j=1}^{t} \Delta LnREX_J^{+} = \sum_{j=1}^{t} \max\left(\Delta LnREX_{j,} 0\right),$$

$$NEG_t = \sum_{j=1}^{t} \Delta LnREX_J^{-} = \sum_{j=1}^{t} \min(\Delta LnREX_J, 0) \qquad (3)$$

POS_t represents the partial total of currency appreciation, whereas NEG_t represents the partial sum of currency devaluation. By replacing it to the linear model, we arrive at the nonlinear model formulation below.

$$\Delta LnTB_{i,t} = \acute{a} + \sum_{j=1}^{n1} \acute{b}_j \, \Delta LnTB_{i,t-j} + \sum_{j=0}^{n2} \acute{c}_j \Delta LnY_{PAK,t-j} + \sum_{j=0}^{n3} \acute{d}_j \Delta LnY_{Chn,t-j} + \sum_{j=0}^{n4} \acute{e}_j \, \Delta POS_{t-j}$$
$$+ \sum_{j=0}^{n5} \acute{f}_j \, \Delta NEG_{t-j} + \theta_0 LnTB_{i,t-1} + \theta_1 LnY_{PAK,t-1} + \theta_2 LnY_{Chn,t-1} + \theta_3 POS_{t-1}$$
$$+ \theta_4 NEG_{t-1} + \epsilon_t \qquad (4)$$

Equation 4 illustrates the nonlinear model description, as contrasted to Equation 2's linear model formulation. Shin et al. (2014) proposed that the critical values given by (Pesaran et al., 2001) for the linear model may be employed in verifying the Cointegration in the instance of the nonlinear model given in Equation 4. Due to the reliance of the two partial sum variables, F-test allows us to employ the very same parameter estimates.

Equation 4 estimates propose an opportunity to test the symmetry of the exchange rate along with several dimensions. For instance, ΔPOS and ΔNEG different lag structures put light on the currency appreciation versus depreciation short-run adjustment asymmetry on the trade balance. Similarly, we get to know the differences in the direction and magnitude of currency appreciation and depreciation effect on the trade balance by the estimates of ΔPOS and ΔNEG. The existence of impact asymmetry or short-run cumulative can be confirmed by rejecting a null $H_0: \sum \widehat{e'}_j = \sum \widehat{f'}_j$ finally, the asymmetric effects of RBER, in the long run, can be confirmed if the null $H_0: -\hat{\theta}_3/\hat{\theta}_0 = \hat{\theta}_4/\hat{\theta}_0$ in favor of inequality.

4. Results and Discussions

In this section, we begin by discussing ADF test results based on which we proceeded to ARDL model. After that, the empirical results of commodity trade between Pakistan and China are reported in Table 2 to 4. Estimation results are carried out through Eviews 10 with a maximum of three optimum lags based on the AIC criterion and sample size.

Table 1: ADF test results of Pakistan and China industry trade

Variables	Level		1st difference		Conclusion
	Intercept	Trend and Intercept	Intercept	Trend and Intercept	
$\mathbf{lnREX_{i,t}}$	0.0165	0.0749	-	-	I(0)
POS	0.6273	0.8157	0.0001	0.0006	I(1)
NEG	0.0451	0.0371	-	-	I(0)
$\mathbf{lnY_{PAK}}$	0.0132	0.7420	-	-	I(0)
$\mathbf{lnY_{CHN}}$	0.2135	0.998	0.0992	0.1371	I(1)
lnAVOF	0.0000	0.0001			I(0)
lnBT	0.0071	0.0022			I(0)
lnCHM	0.0278	0.0804			I(0)
lnCT	0.0000	0.2331			I(0)
lnCMIEF	0.0259	0.0004			I(0)
LnFLA	0.1642	0.0163			I(0)
LnMTE	0.0001	0.0000			I(0)
LnMGC	0.5338	0.8565	0.0001	0.0004	I(1)
LnMFL	0.2725	0.5235	0.0000	0.0000	I(1)
lnMMI	0.1014	0.0454			I(0)

P-values are used for decision making in ADF test.

The variables lnY_{PAK}, $lnREX_{i,t}$, NEG, lnAVOF, lnCHM, lnCMIEF , lnFLA , lnMTE, lnBT, lnMMI and lnCT are stationary at level, i.e. I (0). While lnY_{CHN} (log of the China real GDP), POS, lnMFL and lnMGC are stationary at 1st difference, hence I (1).

Discussions

We begin with the linear model's results, which are very interesting and differ from the previous studies that use aggregated data like the studies of Aftab and Khan (2008), Awan et al. (2009), and Shahbaz et al. (2012) reveal that under the traditional view, the J-curve does not exist since negative values are accompanied by negative correlation coefficient. However, our findings show the deterioration of trade balance at the shorter lag and improvements at higher lags in the short run for two industries coded as 02 and 09, which means that the J curve according to the old definition can only be found in the two out of ten industries. As per the new J curve is concerned, there are two industries out of ten coded as 03 and 04 exhibit this property, i.e., short-run deterioration of trade balance followed by long-run improvement fulfilling the new definition of the J curve given by Rose and Yellen (1989).

Next, we discuss the results of the nonlinear model and see some additional information that was absent in the linear models are now obvious in the nonlinear models. Most of the industries' trade balance shows at least one significant coefficient estimate either for RBER appreciation (ΔPOS) or depreciation (ΔNEG). Except for some industries coded as 01, 06, and 07. In linear models these insights were unseen. As in the linear models, only two Industries coded as 02, 09 show an exchange rate short-run effect. The nonlinear models stand important because it helps us to know and make an assessment of the trade balance adjustment time due to the exchange rate appreciation and depreciation. On the other hand, in linear models, we were not able to assess the adjustment time. Other than the industries coded as 01, 04, 06, and 07, ΔPOS and ΔNEG short-run estimates take different lag lengths which indicate the short-run asymmetry of exchange rate on the balance of trade. Industries coded as 02, 03, 05, 08, 09, and 10 stands as an example of this. In addition, the result shows that based on the sizes and signs, the estimates of both ΔPOS and ΔNEG are different across the sample. This indicates that the rupee appreciations and depreciations have different effects in the short run on the single-industry trade balance as well as the across the group of industries.

Hence, rupee appreciation and depreciation have different effects in the short-run in the case of a single or pool of industries trade balance. These results were again absent in the linear model estimations and we count them in the account of nonlinear models. In the last, Wald statistics show significant values in the case of industries coded as 02, 03, 04, 05, and 10. We report it as Wald-S in the tables. These values support the convincing evidence of impact or short-run commutative asymmetry.

In the next part, we move towards the B and C panels. We see the estimates of industries coded as 02, 03, 04, 05, 08, 09, and 10 supported by Cointegration which together indicate that the rupee appreciation and depreciation short-run effect are translated to long-run effects.

Moreover, industries coded as 03, 04, 05, and 08 which carried indications of deterioration (improvement) in the short run due to the devaluation of the rupee are now, in the long run, carry a significant and positive coefficient for either the POS or the NEG variables. On the other hand, the linear model lacks this pattern except for two industries. This increase in industries' trade balance response is attributed to the non-linear adjustments of the exchange rates. In these four industries' trade dynamics, we take it J-Curve evidence by following Bahmani-Oskooee and Fariditavana (2015). In contrast to these findings of nonlinear models only two industries shown evidence of J-curve. Finally, the estimates of the Wald test as reported as Wald-L in the table show significant values in the industries coded as 03, 04, 05, and 08 which indicate long-run asymmetric effects.

Pakistan and China GDP (Income) are used as a proxy indicator for demand and production capacity in our study. The sign of trading partner (China) income can be negative and positive. The favorable and expected sign is positive because as the income of trading partners increases it will enhance their imports in order to meet the demand they created. As a results, exports of Pakistan will increase to that country and affect trade balance positively. However, it is also possible that this sign may be negative. As the GDP of China grows, it can go for the import substitute goods, which eventually decreases its import.

The result of the China income impact on the industries trade balance is given in Table 2. The coefficients of 3 industries out of 6 are positive and significant at the 10% level of significance. These industries are coded as 02, 06, and 07. In contrast to the linear model, the nonlinear model gave evidence that China's income ($lnYChn$) carries a meaningful positive significant coefficient for 2 industries. On the other hand, there are two industries whose trade balance shows a negative association with the income of China, these industries are coded as 04 and 05. In the nonlinear model, the GDP of China $lnYChn$ holds meaningful negative significance for 5 industries.

The relationship between Pakistan Income and the trade balance of industries can also be positive and negative. The favorable and expected sign is positive. There are three industries coded as 04, 05, and 07 whose trade balance shows a positive correlation with Pakistan income.

There are some industries whose trade balances show negative affiliations with Pakistan income. These industries are coded as 02 and 06. As far as the long-run coefficients under the nonlinear model are concerned, it is found that a significant coefficient of Pakistan’s income has been carried out in 3 industries which are also supported by Cointegration. On the other hand, the linear model gives such evidence in other industries which reflects the fact that nonlinear model results are different than the linear model. Furthermore, in two industries Pakistan Income $Ypak$ estimates carry a negative significant coefficient which are considered as strong evidence to the fact that along with the growth path, the Pakistan economy has the propensity to substitute imports.

To ensure the accuracy of our findings, we used many diagnostic tests in our research. F-test was used to see if there is Cointegration among the variables or not. Based on the test finding, we see that almost all the statistics shows evidence of Cointegration except only one model where the f-test is insignificant, we produce the error term using standardized long-run coefficients estimations and long-run model (1) in this alternative test. We examine the linear error-correction model (2), labeling it error term ECM, and substitute the linear combination of lagged level variables with ECMt-1, estimating the new specification with the same optimal lags. If ECMt-1 has a considerably negative coefficient, Cointegration would be maintained. To verify that there is no autocorrelation in the residual, a Lagrange multiplier statistic with four degrees of freedom was applied. Because there is no autocorrelation, we discovered that the majority of our results are trustworthy. We have used Ramsey's reset test with 1 degree of freedom to make the specifications of the model. Insignificant statistics are discovered to ensure that models are accurately defined. Following (Pesaran et al. (2001) we employ CUSUM and CUSUM square for short and long term coefficient stability. Models having stable and unstable estimates are denoted by 'S' and 'NS,' respectively, in Panel C. Our findings indicate that the majority of our models are stable. R2 was included to ensure goodness of fit.

Table 2: Linear ARDL (L-ARDL) and Nonlinear ARDL (NL-ARDL) Models for Pakistan and China industry trade

	AVOF		B&T		CH		CM & TR	
	LARDL	NLARDL	LARDL	NLARDL	LARDL	NLARDL	LARDL	NLARDL
Panel A (Short–Run Estimates)								
$\Delta lnY_{PAK,t}$	-0.40(3.65)	-0.19(1.50)	-4.15(1.30)	-1.91(1.58)	-0.76(1.32)	0.91(1.36)	5.89(2.09)	7.90(2.44)
$\Delta lnY_{PAK,t-1}$			7.66(0.49)	6.59(0.35)			3.20(2.24)	1.97(3.29)
$\Delta lnY_{PAK,t-2}$			-7.13(2.51)	-4.77(2.03)			-7.32(2.25)	-1.27(2.74)
$\Delta lnY_{PAK,t-3}$			6.31(5.56)	9.16(3.42)				
$\Delta lnY_{i,t}$	0.01(0.38)	0.05(1.23)	2.85(3.66)	9.39(2.55)	1.87(1.78)	-2.17(1.23)	-3.56(5.45)	-5.67(4.28)
$\Delta lnY_{i,t-1}$			-1.73(3.96)	-3.85(1.01)		10.32(2.31)	14.78(4.88)	9.09(4.75)
$\Delta lnY_{i,t-2}$			-2.58(1.72)	-6.81(1.89)		-4.36(2.19)		
$\Delta lnY_{i,t-3}$						1.33(1.53)		
$\Delta lnREX_{i,t}$	0.01(0.39)		-2.01(1.28)		0.17(1.59)		-1.17(1.56)	
$\Delta lnREX_{i,t-1}$			3.82(1.67)		-0.29(1.75)		-0.99(1.94)	
$\Delta lnREX_{i,t-2}$					0.03(0.20)		-0.96(1.23)	
$\Delta lnREX_{i,t-3}$								
ΔPOS_t		0.06(0.92)		1.10(0.27)		0.27(0.57)		-2.64(1.86)
ΔPOS_{t-1}				14.02(1.79)		-0.25(0.50)		
ΔPOS_{t-2}				-2.76(0.53)		-0.21(0.63)		
ΔPOS_{t-3}				-4.30(1.40)		-0.54(2.06)		
ΔNEG_t		-0.02(0.75)		-3.73(0.98)		0.14(0.56)		0.60(0.42)
ΔNEG_{t-1}						-0.85(2.49)		-0.73(0.62)
ΔNEG_{t-2}						0.82(1.43)		-1.97(1.68)
ΔNEG_{t-3}								
Panel B (Long-Run Estimates)								
lnY_{PAK}	-0.03(0.34)	-0.17(1.18)	-3.75(4.03)	-4.35(1.84)	-0.85(1.46)	0.56(1.86)	6.40(2.86)	9.27(2.76)
lnY_i	0.01(0.38)	0.04(1.30)	1.94(3.92)	2.65(1.97)	0.33(1.22)	-0.57(3.06)	-3.41(3.34)	-4.30(3.06)
$lnREX_i$	0.01(0.39)		-9.03(3.93)		0.59(2.60)		4.15(6.16)	
POS		0.05(0.96)		-9.64(3.11)		1.95(6.97)		4.38(4.15)
NEG		-0.02(0.78)		-12.85(1.87)		1.44(8.05)		5.36(4.99)
Constant	0.35(0.28)	2.97(1.59)	436.75(4.15)	662.84(1.75)	10.84(1.41)	1.93(0.57)	-76.83(2.60)	-110.74(2.41)
Panel C (Diagnostic Statistics)								
F test	9.9	7.5	13.07	7.93	5.37	7.4	16.31	15.2
ECM_{t-1}	-1.09(6.68)	-1.13(11.61)	-1.67(5.41)	-1.22(2.69)	-0.89(4.15)	-1.61(3.62)	-0.99(6.06)	-0.96(8.23)
LM	0.80	0.56	0.47	0.23	0.000	0.66	0.99	0.60
RESET	0.00	2.39	5.28	2.35	10.39	2.43	3.36	0.46
Adjusted R^2	0.54	0.55	0.82	0.85	0.63	0.94	0.82	0.90
$CS(CS^2)$	S(NS)	S(NS)	S(S)	S(S)	S(NS)	S(S)	S(S)	S(S)
WALD − S		1.46		22.73		6.87		5.97
WALD − L		0.32		0.43		11.09		11.33

Source: Author(s) calculations
Note: Coefficients are followed by absolute value of t-statistics.

AVOF (Animal and vegetable oils and fats)
B&T (Beverages and tobacco)
CH (Chemical)
CM & TR (Commodities & transacts. Not class. Acc)

Table 3: Linear ARDL (L-ARDL) and Nonlinear ARDL (NL-ARDL) Models for Pakistan and China industry trade

	CM & ID		FD & LA		MC & TE		MG	
	LARDL	NLARDL	LARDL	NLARDL	LARDL	NLARDL	LARDL	NLARDL
Panel A (Short–Run Estimates)								
$\Delta \ln Y_{PAK,t}$	-0.37(1.49)	0.23(0.14)	0.52(0.09)	-5.59(1.13)	0.18(3.63)			
$\Delta \ln Y_{PAK,t-1}$	0.16(2.93)	-4.68(1.44)	-7.43(1.91)					
$\Delta \ln Y_{PAK,t-2}$		-0.16(0.04)	31.86(1.87)	31.08(3.78)	0.02(0.23)		14.90(1.12)	14.78(1.37)
$\Delta \ln Y_{PAK,t-3}$		-2.23(0.86)	-23.44(3.11)	-20.82(5.08)	0.16(1.65)		-11.42(1.64)	-10.80(2.16)
$\Delta \ln Y_{i,t}$	1.90(2.01)	1.89(1.69)			-0.12(1.71)			
$\Delta \ln Y_{i,t-1}$		-1.65(0.41)						
$\Delta \ln Y_{i,t-2}$		-1.24(0.31)	-14.83(3.95)	-7.32(2.04)	0.04(0.76)	0.09(2.11)	4.46(1.96)	6.86(1.86)
$\Delta \ln Y_{i,t-3}$		3.97(1.64)	3.49(0.44)	-14.95(1.36)	0.09(1.09)	0.16(1.34)		-13.51(2.11)
$\Delta \ln REX_{i,t}$	-0.17(1.68)		2.11(0.27)	3.01(0.20)	-0.18(2.94)		-0.30(2.27)	
$\Delta \ln REX_{i,t-1}$	-0.004(0.39)		-9.54(1.65)	-8.53(0.91)			0.08(1.20)	
$\Delta \ln REX_{i,t-2}$	0.69(0.85)		1.22(0.62)					
$\Delta \ln REX_{i,t-3}$			-0.28(0.39)					
ΔPOS_t		-0.69(0.73)		0.04(0.02)		1.82(1.90)		-1.43(0.24)
ΔPOS_{t-1}				-0.09(0.58)				
ΔPOS_{t-2}				4.86(3.33)				
ΔPOS_{t-3}								
ΔNEG_t		-0.61(0.92)		3.00(0.86)		-0.01(0.43)		5.73(1.75)
ΔNEG_{t-1}		1.52(2.20)		-4.27(1.31)				
ΔNEG_{t-2}				-1.01(0.24)				
ΔNEG_{t-3}								
Panel B (Long-Run Estimates)								
$\ln Y_{PAK}$	7.41(2.22)	17.85(5.59)	-2.05(1.89)	-3.47(1.77)	0.23(1.91)	0.24(2.46)	-6.63(1.39)	0.16(0.17)
$\ln Y_i$	-3.68(2.50)	-7.42(5.20)	1.18(2.30)	1.73(2.63)	0.10(1.92)	-0.09(2.05)	2.80(1.34)	-1.19(3.23)
$\ln REX_i$	0.25(0.18)		-0.28(0.71)		0.04(1.22)		-1.01(0.57)	
POS		0.98(0.96)		-0.39(0.67)		-0.02(0.73)		3.40(16.50)
NEG		3.97(5.24)		-0.68(0.88)		0.03(1.30)		1.04(6.10)
Constant	-83.49(1.88)	- 236.83(5.76)	19.86(1.49)	38.96(1.20)	2.95(1.86)	-3.57(2.73)	92.53(1.38)	28.93(2.11)
Panel C (Diagnostic Statistics)								
F test	8	8.41	4.28	3.13	354.24	12.4	1.46	10.35
ECM_{t-1}	-0.77(5.28)	-1.81(4.23)	-0.68(4.92)	-0.68(4.88)	-1.05(28.12)	-1.41(7.73)	-0.17(2.26)	-1.11(7.88)
LM	0.92	1.19	5.3	10.09	1.47	2.07	0.64	0.13
RESET	0.24	0.0009	0.32	0.49	1.48	0.56	0.25	9.95
Adjusted R^2	0.85	0.93	0.57	0.58	0.98	0.84	0.26	0.83
$CS(CS^2)$	S(S)	S(S)	S(S)	S(S)	S(NS)	S(S)	S(S)	S(S)
WALD − S		11.80		0.87		1.02		11.78
WALD − L		21.90		1.09		2.09		10.87

Source: Author(s) calculations
Note: Coefficients are followed by absolute value of t-statistics.

CM & ID (Crude materials, inedible, except f)
FD & LA (Food and live animals)
MC & TE (Machinery and transport equipment)

MG (Manufacture goods classified chiefly b)

Table 4: Linear ARDL (L-ARDL) and Nonlinear ARDL (NL-ARDL) Models for Pakistan and China industry trade

	Mineral fuels, lubricants and relat		Miscellaneous manufactured	
	LARDL	NLARDL	LARDL	NLARDL
Panel A: Short–Run Estimates				
$\Delta \ln Y_{PAK,t}$	4.60(1.13)	-4.04(0.75)	0.46(2.81)	0.28(2.33)
$\Delta \ln Y_{PAK,t-1}$		6.67(0.67)		0.35(1.44)
$\Delta \ln Y_{PAK,t-2}$		7.60(0.92)		0.31(1.69)
$\Delta \ln Y_{PAK,t-3}$		-18.11(2.50)		-0.48(3.73)
$\Delta \ln Y_{i,t}$	-2.28(1.23)	8.57(1.34)	-0.71(3.80)	-0.59(3.74)
$\Delta \ln Y_{i,t-1}$		-6.91(2.07)	0.44(2.62)	0.55(2.29)
$\Delta \ln Y_{i,t-2}$		2.47(3.26)	-0.31(3.37)	-0.46(3.24)
$\Delta \ln Y_{i,t-3}$		-9.15(3.99)		
$\Delta \ln REX_{i,t}$	1.98(1.61)		0.06(1.61)	
$\Delta \ln REX_{i,t-1}$	-2.65(1.48)		-0.002(0.14)	
$\Delta \ln REX_{i,t-2}$	2.40(3.09)		0.03(1.99)	
$\Delta \ln REX_{i,t-3}$			-0.05(4.77)	
ΔPOS_t		5.01(1.96)		-0.04(0.70)
ΔPOS_{t-1}		-8.94(3.72)		0.15(2.06)
ΔPOS_{t-2}		7.28(2.56)		
ΔPOS_{t-3}		4.21(2.02)		
ΔNEG_t		-2.21(1.60)		0.10(2.16)
ΔNEG_{t-1}				-0.12(2.51)
ΔNEG_{t-2}				0.11(4.58)
ΔNEG_{t-3}				
Panel B: Long-Run Estimates				
$\ln Y_{PAK}$	6.70(1.20)	3.88(0.90)	0.07(0.60)	0.05(0.42)
$\ln Y_i$	-3.31(1.31)	-0.79(0.44)	-0.02(0.45)	0.04(0.84)
$\ln REX_i$	0.36(0.29)		0.06(1.51)	
POS		-3.79(5.19)		-0.15(2.52)
NEG		-1.43(1.70)		0.006(0.17)
Constant	-74.88(1.04)	-73.37(1.25)	-1.20(0.75	-2.38(1.50)
Panel C: Diagnostic Statistics				
F test	14.12	18.64	9.42	11.4
ECM_{t-1}	-0.68(3.71)	-1.54(1.37)	-1.02(6.22)	-1.24(9.29)
LM	0.76	0.12	0.36	0.17
RESET	2.55	2.87	1.98	6.13
Adjusted R^2	0.71	0.88	0.78	0.91
$CS(CS^2)$	S(S)	S(S)	S(S)	S(S)
$WALD - S$		10.33		17.21
$WALD - L$		0.56		8.70

Source: Author(s) calculations

Note: Coefficients are followed by absolute value of t-statistics.

MF & LR (Mineral fuels, lubricants and relat)

MM (Miscellaneous manufactured)

5. Conclusions and Policy Implications

The study highlights key policy implications, noting that the elasticity approach to the balance of payments is not highly effective; therefore, policymakers should also consider income-based and monetary approaches. Second, NER (Nominal exchange rate) depreciation does not necessarily translate into RER depreciation. Therefore, a nominal devaluing strategy will indeed be effective only when it transforms towards real devaluation, that could happen unless local prices do not rise sufficiently compared to foreign ones.

Additionally, rather than relying on the external policies intending to improve trade balance; the Government ought to emphasize the supply side that gives a suitable atmosphere so the exportable and import substitute goods can be produced. The findings of the study suggest that sole dependence on external measures such as currency devaluation or depreciation is insufficient to achieve improvements in the trade balance since these countries act as price takers in the global market and lack the ability to influence external demand for the exports of goods by the incentives produced by the prices changes after devaluation. However, Currency depreciation continues to be an important policy tool for improving balance of trade for several sectors in Pakistan-China trade.

It is also suggested that the government should adopt policies that focus on improving Pakistan's real GDP to improve the trade balance. For this reason, the policymakers should adopt such energetic approaches that can promote the agricultural and manufacturing sector's production. To meet foreign and local demand for our goods and services, Government should encourage investment in manufacturing and agricultural sectors through foreign investors so it may improve real GDP and thus industries trade balance.

Future research directions

A further disaggregates analysis based on 2-digits industry between Pakistan and its other trading partners would be a good add up to the literature of international trade.

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APPENDIX

List of Abbreviations

Words	Abbreviations	Industry codes
$\mathbf{lnREX_{i,t}}$	Log of real bilateral exchange rate	-
POS	Positive changes in exchange rates	-
NEG	Negative changes in exchange rates	-
$\mathbf{lnY_{PAK}}$	Log of Pakistan GDP	-
$\mathbf{lnY_{CHN}}$	Log of China GDP	-
lnAVOF	Log of Animal and vegetable oils and fats	01
lnBT	Log of Beverages and tobacco	02
lnCHM	Log of Chemicals	03
lnCT	Log of Commodities & transacts. Not class. Acc	04
lnCMIEF	Log of Crude materials, inedible, except f	05
LnFLA	Log of Food and live animals	06
LnMTE	Log of Machinery and transport equipment	07
LnMGC	Log of Manufacture goods classified chiefly b	08
LnMFL	Log of Mineral fuels, lubricants and relat	09
lnMMI	Log of Miscellaneous manufactured	10